Winners and Losers from Price-Level Volatility: Money Taxation and Information Frictions

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May 12, 2016

Abstract

We analyze an economy with taxes and transfers denominated in dollars and an information friction. It is the information friction that allows for volatility in equilibrium prices and allocations. When the price level is expected to be stable, the competitive equilibrium allocation is Pareto optimal. When the price level is volatile, it is not Pareto optimal, but the stable equilibrium allocations do not necessarily dominate the volatile ones. There can be winners and losers from volatility. We identify winners and losers and describe the effect on them of increases in volatility. Our analysis is an application of the weak axiom of revealed preference in the tax-adjusted Edgeworth box.

1 Introduction

Finance is an important source of efficiency in modern economies, but it is also a source (perhaps *the* major source) of excess economic volatility, i.e., the

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potential for volatility of economic outcomes beyond the volatility of the economic fundamentals. Securities and contracts that pay off in dollars or taxes and transfers fixed in dollars can be sources of proper sunspot equilibrium outcomes.

In our model, lump-sum money taxes are set before the price level is known and expectations are formed.[1] The taxes are exogenous. The policy maker sets money taxes and the agents form expectations. Given these, there is an equilibrium outcome. In equilibrium, the price expectations of the agents must be consistent with the outcomes: rational expectations obtain. The price level is sunspot-driven. The set of instruments is incomplete: sunspot-dependent money taxation is assumed to be unavailable to the government. Nominal taxes do not depend on the realization of sunspots, but real taxes do.[2]

There are 3 consumers. The 2 full-information consumers can "see" sunspots and hedge on the securities market against the effects of sunspot-driven price-level volatility. The third consumer is the restricted-information consumer. He cannot see sunspots. He cannot hedge against the effects of price-level volatility: his participation on the securities market is restricted by the information friction.[3] He must raise money in the spot market for paying his dollar tax by selling some of his commodity endowment or, if he is subsidized, use his money subsidy to buy the consumption good in the spot

[1]Our present interpretation is that the government sets money taxes. Hence we have outside money. Another interpretation (due to Neil Wallace) is that what we call taxes and transfers actually represent past private money borrowing and lending, a case of inside money. Either interpretation is okay. The tax interpretation is the better one for our 2 companion papers on endogenous money taxation.

[2]One might think that, in practice, all observed taxes are real taxes. We disagree. Even income taxes are due in dollars this year but based on last year's dollar income. The money taxes in this paper are meant to be suggestive of general issues arising in modern economies, ones with dollar-denominated financial instruments.

[3]Our model is an extension of the exogenous taxation model of Bhattacharya, Guzman, and Shell (1998). We are currently working on volatility and *endogeneous* taxation. We are preparing 2 papers on endogenous money taxation, one on optimal taxation–the other on voting. See Cozzi, Goenka, Kang, and Shell (2015, 2016).

market. He is always hurt by volatility. The full-information consumers trade *ex ante* in the state-contingent Edgeworth box defined by their tax-adjusted endowments.

When the price level is stable, the competitive equilibrium allocation is Pareto optimal. When the price level is volatile, it is not Pareto optimal, but the stable equilibrium allocation does not always dominate the volatile equilibrium allocations. There can be winners as well as losers from volatility. The full-information consumers hedge by trading securities. One of them (but not both of them) can gain enough to be better off than he would have been without volatility.

Our basic tool is the tax-adjusted Edgeworth box in which the full-information agents hedge against price-level volatility. As a group taken together, the full-information agents are harmed by sunspots. Their aggregate tax-adjusted endowment is negatively correlated with the price-level shocks. A simple condition on taxes and transfers ensures that the tax-adjusted endowment of one of the full information agents is positively correlated with the price-level shocks. He can afford to consume his non-sunspot equilibrium consumption, but he chooses another allocation. By the weak axiom of revealed preference, he is better off. He benefits from volatility. He does so by taking on risk from the other full-information agent. Since the total endowment of the full-information agents is negatively correlated with price-level shocks, the other consumer is necessarily worse off.

We are not the first to observe that there can be winners from sunspot volatility. Goenka and Préchac (2006) address the same issues but in another economy, the incomplete financial-markets (GEI) economy of Cass (1992). They provide a condition on the utility function ensuring that there are winners and losers from volatility. They require a sufficiently high precautionary motive. Kajii (2007) extends their results to more general utility functions. In our paper, we display similar results but in an economy with information frictions (Aumann (1987)) or alternatively with some consumers who are restricted from participating in financial markets (Cass and Shell (1983)).

We provide in Proposition 2 conditions on taxes and transfers for one of the full-information consumers to be better off with price-level volatility while the other full-information consumer is worse off. In the proposition, we allow for (1) heterogeneous preferences and (2) utility functions that merely possess positive first derivatives and negative second derivatives. We do not show that expected utilities are monotone in volatility for this general case. We conjecture that monotonicity does not apply generally. Our intuition for this conjecture is based on the possibility in the general case of multiple sunspot equilibria. However, with identical homothetic preferences (in Section 4), there is a representative agent (for the full-information agents) and thus, a unique equilibrium is guaranteed.[4] For the special case of identical CRRA preferences, we show in Proposition 5 that the expected utility of the winner is indeed strictly increasing in volatility while the expected utilities of the losers are strictly decreasing in volatility.

2 The Model

We analyze a simple exchange economy with lump-sum taxes-and-transfers denominated in money units (say dollars), a single commodity (say chocolate), 3 consumers $h = 1, 2, 3$, and 2 sunspots states $s = \alpha, \beta$. The consumption of Mr. h in state s is $x_h(s) > 0$ (measured in chocolate). His endowment of chocolate is independent of s, $\omega_h(\alpha) = \omega_h(\beta) = \omega_h > 0$. His lump-sum dollar tax is also independent of s, $\tau_h(\alpha) = \tau_h(\beta) = \tau_h$. If τ_h is negative, he is subsidized. If τ_h is zero, then he is neither taxed nor subsidized. Mr. h's expected utility is given by

$$V_h = \pi(\alpha) u_h(x_h(\alpha)) + \pi(\beta) u_h(x_h(\beta)),$$

[4] See Chipman (1974) Theorem 3, page 32.

where $\pi(s)$ is the probability of realization $s = \alpha, \beta$. We assume that $u_h' > 0, u_h'' < 0$, and that indifference curves in $(x_h(\alpha), x_h(\beta))$ space do not intersect the axes, thus ensuring interior solutions to the consumer problems.

We assume that the government sets τ_h before expectations are formed and s is realized. The timing is the source of incomplete instruments: $\tau_h(\alpha) = \tau_h(\beta) = \tau_h$. See our time line, Figure 1.

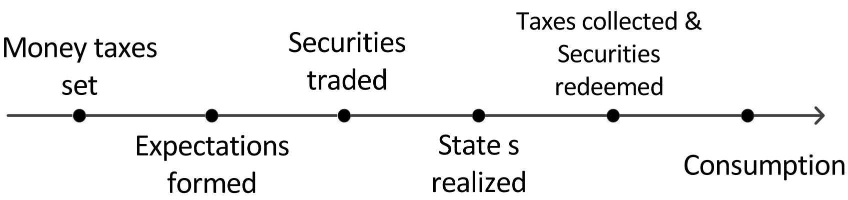

Figure 1: The time line

We restrict attention to the case of balanced taxation,

$$\tau_1 + \tau_2 + \tau_3 = 0.$$

Otherwise the chocolate price of money must be zero[5] and autarky is the only equilibrium.

Let $p(s)$ be the *ex-ante* (accounting) price of chocolate delivered in state s and $p^m(s)$ be *ex-ante* (accounting) price of money delivered in state s. Then $P^m(s) = p(s)/p^m(s)$ is the chocolate price of money in s, while $1/P^m(s)$ is the money price of chocolate in s, or the general price level in s. We assume that consumer 3 is restricted from participation in the securities market because he is blind to sunspots (or for any of many possible other reasons including that he is not born in time to hedge his bets), but consumers 1 and 2 are unrestricted; they see sunspots perfectly. This is a special example of "information frictions" (or correlated, or asymmetric, information).[6]

[5] See Balasko-Shell (1993) on balancedness and bonafidelity.

[6] See Aumann (1987) for the definition of correlated equilibrium in games. See Peck

Consumer 3's problem is simple. He chooses $x_3(s) > 0$ to

$$\text{maximize } u_3\left(x_3(s)\right)$$

subject to

$$p(s)x_3(s) = p(s)\omega_3 - p^m(s)\tau_3$$

for $s = \alpha, \beta$.

Define the tax-adjusted endowment $\widetilde{\omega}_h\left(s\right) = \omega_h - P^m(s)\tau_h$. Then, Mr. 3's budget constraints reduces to

$$x_3(s) = \widetilde{\omega}_3\left(s\right)$$

for $s = \alpha, \beta$. Mr. 3 is passive: he consumes his tax-adjusted endowment in state s.

Mr. 1 and Mr. 2 trade in the securities market and the spot market. Each faces a single budget constraint. Mr. h's problem is to choose $(x_h(\alpha), x_h(\beta)) > 0$ to

$$\text{maximize } V_h$$

subject to

$$p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = \left(p(\alpha) + p(\beta)\right)\omega_h - \left(p^m(\alpha) + p^m(\beta)\right)\tau_h$$

for $h = 1, 2$. From the first-order conditions, we have

$$\frac{p(\beta)}{p(\alpha)} = \frac{\pi\left(\beta\right)u_1'\left(x_1\left(\beta\right)\right)}{\pi\left(\alpha\right)u_1'\left(x_1\left(\alpha\right)\right)} = \frac{\pi\left(\beta\right)u_2'\left(x_2\left(\beta\right)\right)}{\pi\left(\alpha\right)u_2'\left(x_2\left(\alpha\right)\right)}. \tag{1}$$

Market clearing implies

$$x_1(s) + x_2(s) + x_3(s) = \omega_1(s) + \omega_2(s) + \omega_3(s)$$

and Shell (1991) for correlated sunspots in imperfectly competitive market economies. See also Aumann, Peck, and Shell (1985).

or simply

$$x_1(s) + x_2(s) + x_3(s) = \widetilde{\omega}_1(s) + \widetilde{\omega}_2(s) + \widetilde{\omega}_3(s) \tag{2}$$

for $s = \alpha, \beta$. But $x_3(s) = \widetilde{\omega}_3(s)$, so we have

$$x_1(s) + x_2(s) = \widetilde{\omega}_1(s) + \widetilde{\omega}_2(s) \quad \text{for } s = \alpha, \beta. \tag{3}$$

Equation (3) defines the relevant tax-adjusted Edgeworth box (typically a proper rectangular).

In this financial economy, there is a wide range of possible rational beliefs about the price level, generating in turn a wide range of rational, sunspot equilibria. Our goal is to focus on the effects of increased volatility on the behavior of the agents. Hence we focus on economies that can be ranked on volatility. We therefore focus on rational beliefs that are generated as mean-preserving spreads about some non-volatile price level, $P^m(\alpha) = P^m(\beta) = P^m \geq 0$. We measure volatility by the non-negative mean-preserving spread parameter σ defined by

$$P^m(\alpha) = P^m - \frac{\sigma}{\pi(\alpha)}$$

and

$$P^m(\beta) = P^m + \frac{\sigma}{\pi(\beta)},$$

where P^m is the non-sunspot equilibrium chocolate price of dollars and $\sigma \in [0, \pi(\alpha) P^m)$. When $\sigma = 0$, the equilibrium allocations are not affected by sunspots (a non-sunspots equilibrium). When $\sigma > 0$, the economy is a proper sunspots economy. State α is the inflationary state: a dollar buys less chocolate in state α than in state β. State β is the deflationary state: a dollar buys more chocolate in state β than in state α.

Proposition 1 *The non-sunspot-equilibrium ($\sigma = 0$) allocation is Pareto optimal. The proper sunspot-equilibrium allocation ($\sigma > 0$ and $\tau_3 \neq 0$) is not Pareto optimal.*

Proof When $\sigma = 0$, we have $\widetilde{\omega}_h(\alpha) = \widetilde{\omega}_h(\beta) = \omega_h$ for $h = 1, 2, 3$. The tax-adjusted endowments are Pareto optimal because we have

$$\frac{\pi(\beta)\, u_1'(\omega_h)}{\pi(\alpha)\, u_1'(\omega_h)} = \frac{\pi(\beta)\, u_2'(\omega_h)}{\pi(\alpha)\, u_2'(\omega_h)} = \frac{\pi(\beta)\, u_3'(\omega_h)}{\pi(\alpha)\, u_3'(\omega_h)} = \frac{\pi(\beta)}{\pi(\alpha)}.$$

Each consumer consumes his tax-adjusted endowments, i.e., $x_h(s) = \widetilde{\omega}_h(s)$ where $h = 1, 2, 3$ and $s = \alpha, \beta$, and the equilibrium allocations are Pareto optimal.

For $\sigma > 0$ and $\tau_3 > 0$, we assume (for purposes of contradiction) that the equilibrium allocations are Pareto optimal, which would imply

$$\frac{\pi(\beta)\, u_1'(x_1(\beta))}{\pi(\alpha)\, u_1'(x_1(\alpha))} = \frac{\pi(\beta)\, u_2'(x_2(\beta))}{\pi(\alpha)\, u_2'(x_2(\alpha))} = \frac{\pi(\beta)\, u_3'(x_3(\beta))}{\pi(\alpha)\, u_3'(x_3(\alpha))}. \tag{4}$$

Because $\tau_3 > 0$, we have $\widetilde{\omega}_3(\alpha) > \widetilde{\omega}_3(\beta)$ and therefore $x_3(\alpha) > x_3(\beta)$. Because u_h is strictly concave, we have

$$\frac{\pi(\beta)\, u_3'(x_3(\beta))}{\pi(\alpha)\, u_3'(x_3(\alpha))} > \frac{\pi(\beta)}{\pi(\alpha)}. \tag{5}$$

Because $\widetilde{\omega}_3(\alpha) > \widetilde{\omega}_3(\beta)$, from the market clearing condition (see equation (2)) we have

$$\widetilde{\omega}_1(\alpha) + \widetilde{\omega}_2(\alpha) < \widetilde{\omega}_1(\beta) + \widetilde{\omega}_2(\beta). \tag{6}$$

Inequality (6) and the market-clearing condition (see equation (3).) imply that one of the two following inequalities obtains:

$$x_1(\alpha) < x_1(\beta), \tag{7}$$

$$x_2(\alpha) < x_2(\beta). \tag{8}$$

Inequalities (7) and (8) imply

$$\frac{\pi(\beta)\, u_1'(x_1(\beta))}{\pi(\alpha)\, u_1'(x_1(\alpha))} < \frac{\pi(\beta)}{\pi(\alpha)} \quad \text{and} \quad \frac{\pi(\beta)\, u_2'(x_2(\beta))}{\pi(\alpha)\, u_2'(x_2(\alpha))} < \frac{\pi(\beta)}{\pi(\alpha)} \tag{9}$$

respectively. Either inequality in (9) with inequality (5) violates equation (4). The case of $\sigma > 0$ and $\tau_3 < 0$ can be established in like manner. ■

Proposition 1 is in the spirit of Cass-Shell (1983). Although our model is different from Cass-Shell, the proof is similar. Another similarity with Cass-Shell (1983) is that if everyone has full information, sunspots cannot matter. A dis-similarity with Cass-Shell (1983) is that in the money taxation model when τ is not equal to 0 and everyone is blind to sunspots, there is typically a continuum of sunspot equilibria. In Cass-Shell, when every individual is restricted, the sunspot equilibria are randomizations over a *finite* number of certainty equilibria. Our present paper involves taxation in terms of money; Cass-Shell (1983) is a non-financial model.

3 The price level

See Figure 2. Consider the tax-adjusted Edgeworth box for Mr 1 and Mr 2 in the case in which volatility $\sigma > 0$. The dimensions of the box are $(\widetilde{\omega}_1(\alpha) + \widetilde{\omega}_2(\alpha)) \times (\widetilde{\omega}_1(\beta) + \widetilde{\omega}_2(\beta))$. If $\tau_1 + \tau_2 \neq 0$, the Edgeworth box is a proper rectangle with height different from width, so that $p(\alpha)/\pi(\alpha) \neq p(\beta)/\pi(\beta)$. If $\tau_1 + \tau_2 > 0$, then the α-dimension is larger than the β-dimension, $\widetilde{\omega}_1(\alpha) + \widetilde{\omega}_2(\alpha) > \widetilde{\omega}_1(\beta) + \widetilde{\omega}_2(\beta)$, which implies that we have $p(\alpha)/\pi(\alpha) < p(\beta)/\pi(\beta)$ so the total tax-adjusted-endowment of the 2 unrestricted consumers is negatively correlated with the price level.

Lemma 1 *If* $\widetilde{\omega}_1(\alpha)+\widetilde{\omega}_2(\alpha) > \widetilde{\omega}_1(\beta)+\widetilde{\omega}_2(\beta)$*, then* $p(\alpha)/\pi(\alpha) < p(\beta)/\pi(\beta)$.

Proof: (by contradiction) From the first-order conditions, we have

$$\frac{\pi(\alpha)\, u_1'(x_1(\alpha))}{\pi(\beta)\, u_1'(x_1(\beta))} = \frac{\pi(\alpha)\, u_2'(x_2(\alpha))}{\pi(\beta)\, u_2'(x_2(\beta))} = \frac{p(\alpha)}{p(\beta)}. \tag{10}$$

Assume that $p(\alpha)/\pi(\alpha) \geq p(\beta)/\pi(\beta)$. This implies that $u_1'(x_1(\alpha)) \geq u_1'(x_1(\beta))$ and $u_2'(x_2(\alpha)) \geq u_2'(x_2(\beta))$ by equation (10). Because u_h is

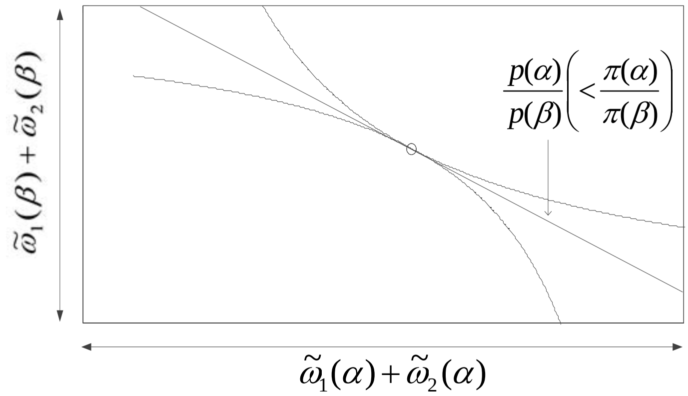

Figure 2: Tax-Adjusted Edgeworth Box

strictly concave, we know that $x_1(\alpha) \leq x_1(\beta)$ and $x_2(\alpha) \leq x_2(\beta)$. This implies that

$$x_1(\alpha) + x_2(\alpha) \leq x_1(\beta) + x_2(\beta). \tag{11}$$

By the market clearing conditions, $x_1(\alpha) + x_2(\alpha) = \widetilde{\omega}_1(\alpha) + \widetilde{\omega}_2(\alpha)$ and $x_1(\beta) + x_2(\beta) = \widetilde{\omega}_1(\beta) + \widetilde{\omega}_2(\beta)$. Because $\widetilde{\omega}_1(\alpha) + \widetilde{\omega}_2(\alpha) > \widetilde{\omega}_1(\beta) + \widetilde{\omega}_2(\beta)$, the inequality (11) violates the market clearing conditions. ■

When is there a winner in the sunspots economy? Since the total tax-adjusted endowment for Mr 1 and Mr 2 is negatively correlated with the price level, a larger tax-adjusted endowment in state s decreases the price $p(s)$ in that state. Even though the total tax-adjusted endowment of the full-information consumers is negatively correlated with the price-level, some consumers' tax-adjusted endowment can be positively correlated with the price level. One possible case is that Mr. 1's nominal tax is larger than Mr. 2's nominal subsidy. In this situation, Mr. 2 can increase his wealth and his expected utility due to volatility by taking on some of Mr. 1's risk. This can be established by the weak axiom of revealed preference; see Figure 3. As σ increases, the tax-adjusted endowment moves from A to B along the dotted line. The dotted line, whose slope is given by the ratio

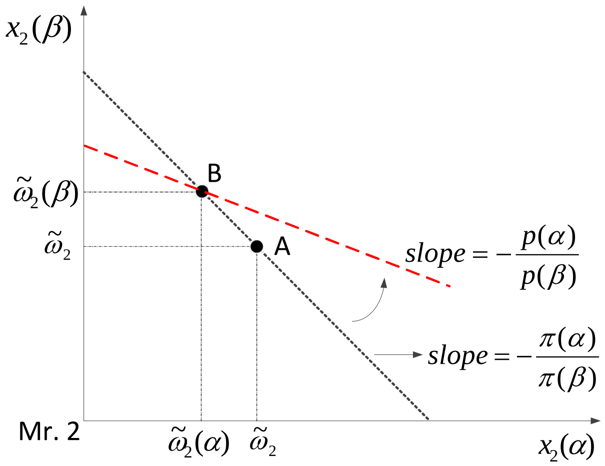

Figure 3: The case of $\tau_2 < 0$

of the probabilities, can be interpreted as (1) the budget line in the non-sunspots economy and also as (2) the set of mean-preserving spreads about the certainty endowment. A is the (unadjusted) endowment. (A is also the equilibrium allocation in the certainty economy.) B is the tax-adjusted endowment and the dashed line represents the budget line for the sunspots economy. In Figure 3, the certainty equilibrium allocation A is affordable in the budget set of the sunspots economy. Therefore, by WARP, Mr. 2's expected utility in the sunspots economy is higher than it is in the certainty economy (because he can afford A, but he chooses B).

Proposition 2 *If* $\tau_1 + \tau_2 > 0$ (< 0) *and* $\tau_2 \leq 0$ (≥ 0)*, Mr 2. is better off with price volatility and Mr. 1 and Mr. 3 are worse off with price volatility.*

Proof: Case 1: $\tau_1 + \tau_2 > 0$ and $\tau_2 \leq 0$

Utility functions are strictly concave and hence Mr. 3 is obviously worse off from price volatility because his equilibrium allocations are the same as his tax-adjusted endowments, which are (by construction) mean-preserving spreads of the non-sunspots allocation.

Mr. 2's non-sunspot equilibrium allocation is $(x_2(\alpha), x_2(\beta)) = (\widetilde{\omega}_2, \widetilde{\omega}_2)$ where $\widetilde{\omega}_2 = \omega_2 - P^m \tau_h$. We need to show that $(\widetilde{\omega}_2, \widetilde{\omega}_2)$ is affordable in the proper sunspots economy. Then, by the WARP, Mr. 2 would be better off with the sunspots allocation.

The condition that $(\widetilde{\omega}_2, \widetilde{\omega}_2)$ is affordable in the sunspots economy is

$$p(\alpha)\widetilde{\omega}_2 + p(\beta)\widetilde{\omega}_2 \leq p(\alpha)\widetilde{\omega}_2(\alpha) + p(\beta)\widetilde{\omega}_2(\beta), \tag{12}$$

where $p(s)$ is ex-ante price of commodity in state s.

In the case where $\tau_2 < 0$, we have $\widetilde{\omega}_2(\alpha) < \widetilde{\omega}_2(\beta)$. By $\widetilde{\omega}_2(\alpha) < \widetilde{\omega}_2(\beta)$ and $\pi(\alpha)\widetilde{\omega}_2(\alpha) + \pi(\beta)\widetilde{\omega}_2(\beta) = \widetilde{\omega}_2$, inequality (12) is equivalent to

$$\frac{p(\alpha)}{\pi(\alpha)} \leq \frac{p(\beta)}{\pi(\beta)}. \tag{13}$$

In the case where $\tau_2 = 0$, inequality (13) is not sufficient to make Mr. 2 better off with volatility because Mr. 2's non-sunspot-equilibrium allocation $(\widetilde{\omega}_2, \widetilde{\omega}_2)$ still lies on the budget line in the sunspots economy. See Figure 4. Therefore, we need another condition, namely that the slope of indifference curve at $(\widetilde{\omega}_2, \widetilde{\omega}_2)$ is different from the slope of the budget line in the sunspots economy. The slope of the indifference curve is $-\pi(\alpha)/\pi(\beta)$ and the slope of the sunspots budget line is $-p(\alpha)/p(\beta)$. Therefore, the condition is

$$\frac{p(\alpha)}{\pi(\alpha)} \neq \frac{p(\beta)}{\pi(\beta)}. \tag{14}$$

Merging inequalities (13) and (14), we have

$$\frac{p(\alpha)}{\pi(\alpha)} < \frac{p(\beta)}{\pi(\beta)}, \tag{15}$$

which is the sufficient condition for Mr 2 being better off with volatility. Inequality (15) is proven in Lemma 1.

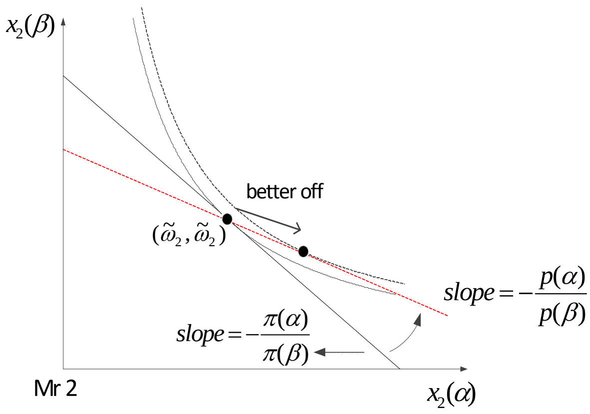

Figure 4: The case of $\tau_2 = 0$

Mr. 1: Given strictly positive prices, $p(\alpha)$ and $p(\beta)$, there are two cases;

$$(a) \quad x_1(\alpha) > \widetilde{\omega}_1(\alpha) \text{ and } x_1(\beta) < \widetilde{\omega}_1(\beta), \tag{16}$$

$$(b) \quad x_1(\alpha) < \widetilde{\omega}_1(\alpha) \text{ and } x_1(\beta) > \widetilde{\omega}_1(\beta). \tag{17}$$

In case (b), Mr. 1 will necessarily be worse off with volatility because of WARP: The equilibrium allocation $(x_1(\alpha), x_1(\beta))$ is affordable with the prices in the non-sunspots economy. (See Figure 5.) Assume by contradiction that case (a) is correct. Then, by the market-clearing conditions, we have

$$x_2(\alpha) < \widetilde{\omega}_2(\alpha) \text{ and } x_2(\beta) > \widetilde{\omega}_2(\beta). \tag{18}$$

Because $\widetilde{\omega}_2(\alpha) < \widetilde{\omega}_2(\beta)$, inequality (18) implies that $x_2(\alpha) < x_2(\beta)$. Therefore, we have

$$\frac{u_2'(x_2(\alpha))}{u_2'(x_2(\beta))} > 1, \tag{19}$$

By equation (10), inequality (19) implies that

$$\frac{p(\alpha)}{\pi(\alpha)} > \frac{p(\beta)}{\pi(\beta)},$$

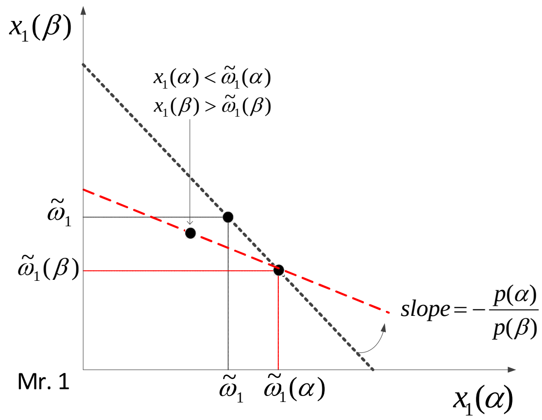

Figure 5: Mr. 1 is worse off with volatility

which violates inequality (15).

Case 2: $\tau_1 + \tau_2 < 0$ and $\tau_2 \geq 0$: This can be established as in Case 1.

■

Proposition 2 shows that in the case where the sign of $\tau_1 + \tau_2$ is different from the sign of τ_2, Mr 2 is better off with the volatile allocation while Mr 1 and Mr 3 are worse off. With the same logic, we can also show that if the sign of $\tau_1 + \tau_2$ is different in sign from τ_1, Mr 1 is better off with volatility, while Mr 2 and Mr 3 are worse off.

Because of balancedness of the tax-transfer plans, the sign of $\tau_1 + \tau_2$ is always opposite to that of τ_3, if τ_3 is not zero. Therefore, both (1) "$\tau_1 + \tau_2 > 0$ and $\tau_2 < 0$" or (2) "$\tau_1 + \tau_2 < 0$ and $\tau_2 > 0$" imply that $sign(\tau_2) = sign(\tau_3)$. The following corollary summarizes this.

Corollary 1 *If Mr h and Mr 3 are both taxed (or both subsidized) where $h \neq 3$, Mr h is better off with volatility and the other two consumers are worse off.*

Proof Directly from Proposition 2.

Remark 1 Note that one of the full-information consumers, say Mr. 2. without any loss of generality, who is receiving a subsidy is still worse off. There are four different effects: a price effect (related to magnitude of $\tau_1+\tau_2$), a direct loss of expected utility from increased volatility from risk averseness, a trade effect as the post-tax endowment moves further away from the minor diagonal of the post-tax Edgeworth box, and the gain from the subsidy (since $\tau_2 < 0$). Corollary 1 says that the first three effects can outweigh the third effect. This is reminiscent of the transfer paradox (see the formulation in Balasko (1978)) where the welfare reversal depends on both the change in prices and the size of the net trade. However, our result is different from the classical transfer paradox as we hold the nominal taxes and transfers constant, and the change in price volatility induces the change in the real taxes and transfers. If there were no price volatility, then $p(\alpha) = p(\beta)$ and Mr. 2 would be unambiguously better off.

The following corollary summarizes how the 3 consumers' expected utilities change with price volatility.

Corollary 2 *The following table summarizes the pattern of winners and losers from price volatility:*

	Full-information consumers		*Restricted-information consumer*	*Full-information consumers*		*Restricted-information consumer*
	Mr. 1	*Mr. 2*	*Mr. 3*	*Mr. 1*	*Mr. 2*	*Mr. 3*
Case 1	*S*	*T or 0*	*T*	*L*	***W***	*L*
Case 2	*T*	*S or 0*	*S*	*L*	***W***	*L*
Case 3	*T or 0*	*S*	*T*	***W***	*L*	*L*
Case 4	*S or 0*	*T*	*S*	***W***	*L*	*L*

S denotes subsidized ($\tau_h < 0$)*, T denotes taxed* ($\tau_h > 0$)*, 0 denotes nei-*

ther subsidized nor taxed ($\tau_h = 0$)*, W denotes winner from volatility, and L denotes loser from volatility.*

Proof Cases 1-4 follows directly from the proof of Proposition 2.

4 CRRA Preferences and Global Analysis

We assume in this section that preferences are identical CRRA. We provide the analysis of individual expected utilities as functions of volatility. The main questions are:

(1) Does increasing σ increase the ratio $p(\beta)/p(\alpha)$? (Proposition 3)

(2) Does a higher CRRA risk aversion parameter ρ make the inter-state price ratio more sensitive to money price volatility? (Proposition 4)

(3) Does increasing volatility σ increase the welfare of winners and decrease the welfare of losers? (Proposition 5)

For identical CRRA preferences, we establish that the answer for each of these 3 questions is "yes". Assume that each of the 3 consumers has CRRA preferences given by

$$\begin{aligned} u(x) &= \frac{x^{1-\rho}}{1-\rho} \qquad \text{when } \rho \neq 1 \\ &= \log x \qquad \text{when } \rho = 1, \end{aligned}$$

where ρ is the relative-risk-aversion parameter, i.e., $\rho = -xu''/u' > 0$.

Proposition 3 *Since the 3 consumers have identical CRRA preferences, as* σ *increases, we have that*

$$\frac{p(\beta)/\pi(\beta)}{p(\alpha)/\pi(\alpha)}$$

increases (decreases) when $\tau_1 + \tau_2 > 0$ (< 0).

Proof: Case 1: $\tau_1 + \tau_2 > 0$

From equations (1) and (3), we have

$$\left(\frac{x_1(\beta)}{x_2(\alpha)}\right)^{-\rho} = \left(\frac{\widetilde{\omega}_1(\beta) + \widetilde{\omega}_2(\beta) - x_1(\beta)}{\widetilde{\omega}_1(\alpha) + \widetilde{\omega}_2(\alpha) - x_2(\alpha)}\right)^{-\rho}. \tag{20}$$

Equation (20) implies that

$$\frac{x_1(\beta)}{x_2(\alpha)} = \frac{\widetilde{\omega}_1(\beta) + \widetilde{\omega}_2(\beta)}{\widetilde{\omega}_1(\alpha) + \widetilde{\omega}_2(\alpha)}. \tag{21}$$

From equations (21) and (1), we have

$$\frac{p(\beta)}{p(\alpha)}\frac{\pi(\alpha)}{\pi(\beta)} = \left(\frac{\widetilde{\omega}_1(\beta) + \widetilde{\omega}_2(\beta)}{\widetilde{\omega}_1(\alpha) + \widetilde{\omega}_2(\alpha)}\right)^{-\rho} \tag{22}$$

Equation (22) is equivalent to

$$\frac{p(\beta)}{p(\alpha)}\frac{\pi(\alpha)}{\pi(\beta)} = \left(\frac{\omega_1 + \omega_2 - P^m(\beta)(\tau_1 + \tau_2)}{\omega_1 + \omega_2 - P^m(\alpha)(\tau_1 + \tau_2)}\right)^{-\rho},$$

which in turn is equivalent to

$$\begin{aligned}
\log\left(\frac{p(\beta)}{p(\alpha)}\frac{\pi(\alpha)}{\pi(\beta)}\right) &= -\rho\log(\omega_1 + \omega_2 - P^m(\beta)(\tau_1 + \tau_2)) \\
&\quad +\rho\log(\omega_1 + \omega_2 - P^m(\alpha)(\tau_1 + \tau_2)) \\
&= -\rho\log\left(\omega_1 + \omega_2 - \left(P^m + \frac{\sigma}{\pi(\beta)}\right)(\tau_1 + \tau_2)\right) \\
&\quad +\rho\log\left(\omega_1 + \omega_2 - \left(P^m - \frac{\sigma}{\pi(\alpha)}\right)(\tau_1 + \tau_2)\right)
\end{aligned} \tag{23}$$

Implicitly differentiating equation (23) with respect to σ, we have

$$\frac{d\log\frac{p(\beta)}{p(\alpha)}}{d\sigma}=\rho\times\underbrace{\left\{\frac{1/\pi\left(\beta\right)}{\frac{\omega_1+\omega_2}{\tau_1+\tau_2}-\left(P^m+\frac{\sigma}{\pi(\beta)}\right)}+\frac{1/\pi\left(\alpha\right)}{\frac{\omega_1+\omega_2}{\tau_1+\tau_2}-\left(P^m-\frac{\sigma}{\pi(\alpha)}\right)}\right\}}_{\text{Positive}}>0. \tag{24}$$

Case 2: $\tau_1+\tau_2<0$: We establish this as for Case 1. ■

Equation (24) shows that as σ is increased, the interstate price ratio increases. The higher is risk-aversion ρ, the higher is the rate of increase in the interstate price ratio $p(\beta)/p(\alpha)$. The inter-state commodity price ratio deviates more from its benchmark certainty equilibrium price when either σ, or ρ, or both is increased.

Proposition 4 *If the 3 consumers have identical CRRA preferences, the greater the risk-aversion parameter ρ, the greater is the rate of increase (decrease) of the price ratio $p(\beta)/p(\alpha)$ for $\tau_1+\tau_2>0$ (<0).*

Proof*: Directly from equation (24).*

Proposition 5 *If the consumers have identical CRRA preferences, the expected utility of the winner is strictly increasing in σ and the expected utilities of the losers are strictly decreasing in σ. The winner and the full-information loser are determined by the conditions in Proposition 2 or Corollaries 1 and 2.*

Proof: Case 1: $\tau_1+\tau_2>0$ and $\tau_2\leq 0$
The Lagrangian is

$$\begin{aligned}L &= \pi\left(\alpha\right)u(x_h(\alpha))+\pi\left(\beta\right)u(x_h(\beta)) \\ &\quad +\lambda\left\{\widetilde{\omega}_h\left(\alpha\right)+\frac{p(\beta)}{p(\alpha)}\widetilde{\omega}_h\left(\beta\right)-x_h\left(\alpha\right)-\frac{p(\beta)}{p(\alpha)}x_h\left(\beta\right)\right\}.\end{aligned}$$

By the envelope theorem, $dV_h/d\sigma$ is

$$\frac{dV_h}{d\sigma} = \lambda \left\{ \frac{\partial \widetilde{\omega}_h(\alpha)}{\partial \sigma} + \frac{p(\beta)}{p(\alpha)} \frac{\partial \widetilde{\omega}_h(\beta)}{\partial \sigma} + \frac{d\left(\frac{p(\beta)}{p(\alpha)}\right)}{d\sigma} \left(\widetilde{\omega}_h(\beta) - x_h(\beta)\right) \right\}. \tag{25}$$

We have $d\left(p(\beta)/p(\alpha)\right)/d\sigma > 0$ from Proposition 4.

For Mr 1, we have

$$\frac{\partial \widetilde{\omega}_1(\alpha)}{\partial \sigma} + \frac{p(\beta)}{p(\alpha)} \frac{\partial \widetilde{\omega}_1(\beta)}{\partial \sigma} = \frac{\tau_1}{\pi(\alpha)} - \frac{p(\beta)}{p(\alpha)} \frac{\tau_1}{\pi(\beta)} < 0,$$

because $p(\beta)/p(\alpha) > \pi(\beta)/\pi(\alpha)$ from the proof of Proposition 2 and $\tau_1 > 0$. We know that $\widetilde{\omega}_1(\beta) - x_1(\beta) < 0$ from the proof of Proposition 2. Therefore, we have $dV_1/d\sigma < 0$ from equation (25).

For Mr 2, we have

$$\frac{\partial \widetilde{\omega}_2(\alpha)}{\partial \sigma} + \frac{p(\beta)}{p(\alpha)} \frac{\partial \widetilde{\omega}_2(\beta)}{\partial \sigma} \geq 0,$$

because $p(\beta)/p(\alpha) > \pi(\beta)/\pi(\alpha)$ and $\tau_2 \leq 0$. We know that $\widetilde{\omega}_2(\beta) - x_2(\beta) > 0$ from the proof of Proposition 2. Therefore, we have $dV_2/d\sigma > 0$ from equation (25).

Case 2: $\tau_1 + \tau_2 < 0$ and $\tau_2 \geq 0$: We establish this as in Case 1. ■

5 Numerical Example

In this section we compute a family of numerical examples. Mr 1 is rich. Mr 2 and Mr 3 each have middle class endowments, but only Mr 3 suffers from the information friction.

$$\omega = (\omega_1, \omega_2, \omega_3) = (116, 100, 100)$$

$$\tau = (\tau_1, \tau_2, \tau_3) = (1, -0.5, -0.5)$$

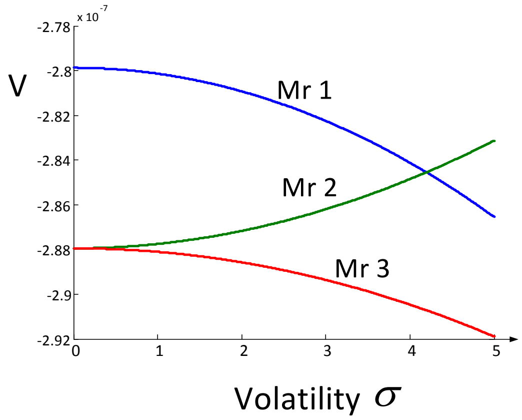

Figure 6: Expected utilities as functions of volatility for the case of $\rho = 4$

This is an example of Case 1 taxation since $\tau_1 + \tau_2 = 1 - 0.5 = 0.5 > 0$ and $\tau_2 = -0.5 < 0$.

Utilities are identical CRRA with risk aversion $\rho > 0$.

$$
\begin{aligned}
u &= \frac{c^{1-\rho}}{1-\rho} \qquad \text{for } \rho \neq 1 \\
&= \log c \qquad \text{for } \rho = 1
\end{aligned}
$$

We assume that the 2 sunspot states are assumed to be equally probable, i.e.,

$$\pi(\alpha) = \pi(\beta) = 0.5.$$

The family of mean-preserving spreads is defined by

$$P^m(\alpha) = P^m - \frac{\sigma}{\pi(\alpha)}$$

$$P^m(\beta) = P^m + \frac{\sigma}{\pi(\beta)},$$

where $P^m = 10$ and $\sigma \in [0, 5)$.

Mr 1 is rich and heavily taxed. He has full information. His expected

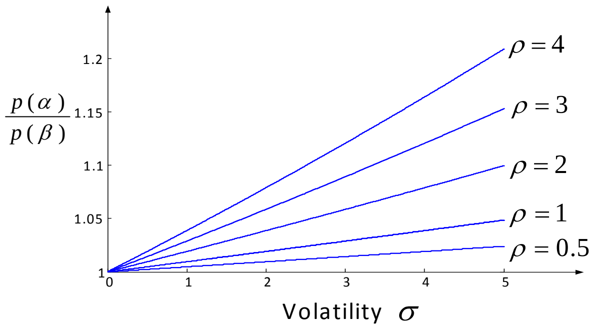

Figure 7: The inter-state price ratio as a function of volatility σ for different values of risk aversion ρ

utility V_1 is strictly declining in volatility σ. Mr 2 and Mr 3 have the same endowments, but Mr 2 has full information while Mr 3 receives no sunspot information. Mr 2's expected utility V_2 is strictly increasing in σ. V_3 is strictly decreasing in σ. See Figure 6, which illustrates Proposition 5. Given risk aversion ρ, the inter-state commodity price ratio is linear in volatility σ. The effect of volatility is amplified as ρ is increased. See Figure 7, which illustrates Propositions 3 and 4.

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