Commitment, Myopia, and Savings

Minwook Kang*

February 16, 2021

Abstract

Previous research in hyperbolic discounting has focused on the role of *illiquid* assets as commitment tools. However, this paper shows that due to an income effect, the consumer has an incentive to use *liquid* financial assets as commitment devices to resolve her self-control problem. The lower the intertemporal elasticity-of-substitution and the more sophisticated the consumer, the higher the commitment demand. A steady-state analysis shows that commitment demand accounts for 29% of the value of the financial asset in Laibson's quasi-hyperbolic discounting model.

Keywords: Commitment; myopia; quasi-hyperbolic discounting; liquid asset; consumption-savings decision

JEL classifications: D9, E2

*Minwook Kang is in the Division of Economics, School of Social Sciences, at Nanyang Technological University (e-mail: mwkang@ntu.edu.sg). I appreciate helpful comments from Lawrence Jin, Lei Wang, Lei Sandy Ye, and the seminar participants at Nanyang Technological University. I thank TingYi Feng for her excellent assistance. All typos and errors are mine.

1. Introduction

Economic agents use a variety of commitment tools to manage their self-control problems. One of the most successful models to explain agents' commitment incentive is quasi-hyperbolic time preferences, developed by Strotz (1956), Phelps and Pollak (1968), and Laibson (1997).[1] In particular, Laibson (1997) investigates the role of illiquid assets as a commitment tool and shows that this commitment tool can improve the welfare of the economy.[2] Since Laibson (1997), there has been considerable research investigating the role of commitment tools, but similar to Laibson (1997), most papers have focused only on illiquid assets as commitment tools[3], and to the best of my knowledge, the role of liquid assets as a commitment tool has not been considered in the literature.[4] This could be due to a common belief that the return on liquid assets is realized in the near future period, so the return can be directly connected with overconsumption but cannot be transferred to wealth in the distant future. However, previous literature has mostly overlooked the income effect, in which today's high savings increase future financial income and thus future savings. Through the income effect, today's savings can contribute to future wealth, which implies that today's savings can be a commitment to future wealth and consumption. Theoretically and quantitatively, we show that this commitment incentive through

[1] Present-biased time preferences have been the subject of extensive research in psychology and economics for several decades. Empirical evidence has suggested that animal and human behavior is short-term oriented and that their discount functions are closer to hyperbolic than exponential. See Thaler (1981), Ainslie and Haendel (1983) and Loewenstein and Prelec (1992). Strotz (1956), Phelps and Pollak (1968), Pollak (1968), Goldman (1979), Laibson (1997) and O'Donoghue and Rabin (1999) apply the theory of quasi-hyperbolic discounting to consumption-savings decision problems.

[2] In a complete financial market (i.e., there exist liquid assets in which returns are equal to or higher than the return on illiquid assets), the illiquid asset cannot be the commitment tool because the consumer can borrow against the illiquid asset by borrowing against the liquid assets.

[3] There is considerable literature related to commitment incentives under time-inconsistent preferences. After Strotz (1956) first formalizes the value of commitment devices under time-inconsistent preferences, Laibson (1997) introduces welfare-improving commitment illiquid assets under hyperbolically-discounted preferences in an incomplete financial market. Gul and Pesendorfer (2001, 2004a, 2004b) and Dekel, Lipman, and Rustichini (2001, 2009) provide the axiomatic foundation for time-inconsistent preferences in which commitment devices are valued. Béabou and Tirole (2002, 2004) show that self-memory selection or self-deception can be used as commitment devices under quasi-hyperbolic discounting. Amador et al. (2006) introduce a time-inconsistent model where the individual faces unverifiable taste shocks. In their model, they showed a partial commitment contract that guarantees that both flexibility and commitment incentives are optimal.

[4] A liquid asset in this paper represents savings in the typical consumption-savings model. To distinguish that from illiquid commitment tools, which are frequently mentioned in the hyperbolic discounting literature, we often use the term liquid assets instead of savings.

the income effect can have a substantial impact on consumers' savings decisions.

For liquid assets to be a commitment device in the consumption-savings decision model, the following two conditions should be satisfied. First, the consumer should be willing to pay for the device more than its direct benefit (marginal utility). Second, the device should affect the future self's budget set. This paper shows that the two conditions are satisfied with liquid assets, so it can be considered a commitment device in a quasi-hyperbolic discounting model. We explain the mechanism of how liquid savings can be used as a commitment tool in a typical three-period consumption-savings model. The intuition for the existence of a positive commitment incentive from liquid savings is as follows. The consumer in the current period (period 1) with self-control problems knows that her future self (period 2) will save too little. However, if the period-2 self has higher financial income, she will save more through the income effect. Therefore, the period-1 current self will have a commitment incentive to save more to increase period-2 financial income, which will induce the period-2 self to save more.[5]

This income effect shows a clear logic of how the liquid asset can be a commitment tool. Nevertheless, this commitment incentive has been ignored in the literature, which might be that previous β-δ quasi-hyperbolic model is constructed based on the time invariance of the value of β. In the β-δ model, the current self's hyperbolic discounting factor (β) is the myopia parameter. However, from the current self's perspective, the future self's discounting factor (β) is unrelated to the current self's preferences, but represents the future self's self-control problem. To distinguish a myopic preference from a self-control problem, we allow the hyperbolic discounting parameters to be time variant. Specifically, in the three-period model, the hyperbolic discounting parameter for the period-1 self is β_1 but the period-2 self's parameter is β_2. With this setting, it is a natural outcome that as β_1 decreases (i.e., as the consumer behaves more myopically), savings decrease. However, this paper shows that the impact of β_2 on the first-period savings is the opposite to that of β_1.

For the current self, the lower value of β_2 means that the future self (period-2 self) has more severe self-control problems. When the current self expects a future self-control problem, which results in low levels of period-2 savings, she will have a commitment incentive to increase period-1 savings. Increased period-1 savings will

[5]This income effect also increases period-2 consumption, which is not a favorable situation for the period-1 self. Therefore, the liquid asset is a less perfect commitment tool than illiquid assets.

increase her period-2 financial income, which has a positive impact on period-2 savings through the income effect. Specifically, our example in section 2 shows that period-1 savings under myopic discounting (i.e., $\beta_1, \beta_2 < 0$) can be even higher than that of exponential discounting (i.e., $\beta_1, \beta_2 = 1$), which is very different from the conventional understanding of an undersavings problem. This paper shows that the high savings with myopic discounting are brought by the commitment incentive (due to low values of β_2) instead of myopia (due to low values of β_1.)[6]

The distinction between myopia and commitment effects is well understood from the Euler equation. We modified the Euler equation to present an inverse savings demand function. This paper indicates that if $\beta_2 < 1$ (the consumer has a self-control problem), the Euler equation has two terms: one (myopic effect) is the same as the typical time-consistent Euler equation and the other (commitment effect) is the additional term from the self-control problem. Both terms have a positive impact on savings demand but the second term (commitment effect) disappears if the consumer has no self-control problem (i.e., $\beta_2 = 1$).

The commitment inventive in this paper could look similar to the sophistication effect analyzed in O'Donoghue and Rabin (1999) but it does not. They show that sophistication mitigates procrastination but exacerbates preproperation in their discrete-choice model. Applying their results to the consumption-savings model, it should be concluded more savings are realized when there is a higher degree of a self-control problem. However, that could be incorrect in the consumption-savings model as shown in this paper. O'Donoghue and Rabin's (1999) model has some limitations in understanding the consumption-savings behavior under quasi-hyperbolic discounting. Specifically, the decision in O'Donoghue and Rabin (1999) is discrete (do it now or delay), which does not directly affect the payoffs in the following periods. However, in the consumption-savings model, increasing savings today increases the financial wealth, which increases the space of subsequent consumption-savings decisions.

Although we use a heterogeneous hyperbolic-discounting model to distinguish the

[6] Based on conventional understanding of the myopia-oriented time-inconsistent problem, the myopic discounting factor always causes undersavings problems. One simple way to evaluate the degree of under-savings is to compare the hyperbolic economy with $\beta < 1$ and the corresponding exponential economy with $\beta = 1$. It is well-known and naturally believed that all periods' savings under hyperbolic discounting are lower than those under exponential discounting. See Harris and Laibson (2001) and Diamond and Koszegi (2003).

two effects, our analysis can also be applied to the steady-state β-δ model. Laibson (1997) introduced the steady-state model to estimate the negative impact of hyperbolic consumption-savings patterns on welfare. In the infinite period model, we derive the Euler equation by assuming that the current self has the discounting factor with β_1 and all future selves have the discounting factor β_2. From the Euler equation of the steady-state, we show that around 29% of the price value of savings can be attributed to a commitment incentive. In this analysis, we assume $\beta = \beta_1 = \beta_2 = 0.7$, which is a commonly used hyperbolic discounting factor in macroeconomics.

The channel of using savings as a commitment device is the income effect. It is well-known that with lower intertemporal elasticity of substitution (IES), the income effect affects the consumer's decision more significantly, and this logic is also applied to our model. A consumer having utility with low IES has a strong desire to balance consumption between the current and future periods. Thus, the increased period-2 income (due to the increased period-1 savings) is more effectively transferred to future wealth by increased period-2 savings. This implies that the commitment value to liquid assets could be higher when the IES is lower. Therefore, this paper shows that as the consumer faces worse a self-control problem (i.e., as β_2 is lower), the period-1 savings will increase if the IES is less than one.

This myopia-commitment analysis can also explain the well-known result that naive and sophisticated choices follow the same savings decisions with log-linear utility function, which is shown by Pollak (1968), Salanié and Treich (2006) and Wei and Selden (2016). Although β_2 represents the degree of self-control problem in this paper, it can also be interpreted as the degree of naivety, as shown in Salanié and Treich (2006). Therefore, we can conclude that as the consumer becomes more sophisticated and the IES becomes lower, the commitment incentive becomes stronger in savings decisions.

The rest of the paper is organized as follows. Section 2 presents a leading example where the demand for savings under myopic discounting can be even higher than that under exponential discounting. Section 3 introduces the three-period consumption-savings model under myopic discounting. Section 4 derives how the myopic and commitment demand for savings are decomposed and derives the inverse savings demand function from the two effects. The analysis under constant IES preferences is conducted in Section 5. The comparison between naive and sophisticated savings decisions are shown in Section 6. Section 7 introduces the steady-state analysis and quantitatively derives the commitment value of savings. Section 8 concludes. All the proofs of propositions are in the Appendices.

2. An example

This section presents a simple example showing how under myopic discounting, the consumer's incentive for self-control plays an important role in her savings decisions. In the popular β-δ model, the savings amount under hyperbolic discounting has been compared to that under exponential discounting with $\beta = 1$. It has been shown that under a constant IES utility function with hyperbolic discounting, the savings amounts in all periods with $\beta < 1$ are always lower than that under exponential discounting with $\beta = 1$ (see Laibson 1997, Harris and Laibson 2001). However, in this example, by allowing the discounting factors to vary across time, the savings in the short run under the time-inconsistent model can be even higher than the corresponding savings in the time-consistent model.

Specifically, in the three period model, we assume that the hyperbolic discounting factor for the period-1 self is β_1 and the hyperbolic discounting factor for the period-2 self is β_2. For the period-1 self, the lower value of β_1 means that the utility value of future consumption is low, so the demand for savings is also low. Therefore, as β_1 decreases, the consumer's savings decrease. However, the impact of β_2 on the savings decision in period 1 is completely different from that of β_1. The low value of β_2 means that the period-1 self faces more severe self-control problems. When β_2 is low, the period-2 self will save less, which results in low period-3 consumption and low welfare for the period-1 self. If the period-1 self expects that the period-2 self will save less (due to low value of β_2), the period-1 self will want to increase period-1 savings, as it will induce high period-2 savings through the income effect. The following example shows this effect.

In this example, we assume that $\delta = 1$ and the gross interest rate is one. The period utility is $u(c) = -c^{-1}$. The hyperbolic discounting factors in periods 1 and 2 are β_1 and β_2, respectively. We assume the consumer has one unit of income in period 1 but no income in periods 2 and 3. In a complete market, the lifetime budget constraint is $c_1 + c_2 + c_3 = 1$. Then, in a complete market, the savings should satisfy $c_1 = 1 - s_1$, $c_2 = s_1 - s_2$ and $c_3 = s_2$. The consumer is fully sophisticated. In period

2, given the savings in period 1, the consumer solves the following problem:

$$\max_{s_2|s_1} u(s_2 - s_1) + \beta_2 u(s_2). \tag{1}$$

From the maximization problem, we have the following savings decision function:

$$\widehat{s}_2(s_1) = \frac{\sqrt{\beta_2}}{1+\sqrt{\beta_2}} s_1. \tag{2}$$

In period 1, the maximization problem with the savings decision function is

$$\max_{s_1} u(1 - s_1) + \beta_1 u(s_1 - \widehat{s}_2(s_1)) + \beta_1 u(\widehat{s}_2(s_1)). \tag{3}$$

From the maximization problem in Eq. (3), the period-1 savings level in terms of (β_1, β_2) is

$$s_1(\beta_1, \beta_2) = \frac{\beta_1 \left(1+\sqrt{\beta_2}\right)^2 - \sqrt{\beta_1 \left(1+\sqrt{\beta_2}\right)^2 \sqrt{\beta_2}}}{\beta_1 \left(1+\sqrt{\beta_2}\right)^2 - \sqrt{\beta_2}}. \tag{4}$$

In the time-inconsistent model where $\beta_1 = \beta_2 = 1$, the optimal savings level in period 1 is $s_1 = 2/3$, as the consumer wants to balance consumption across the three periods.

In the case where $\beta_1 = \beta_2 = \beta$, we have

$$s_1(\beta, \beta) = \frac{\beta^{1/4} + \beta^{3/4}}{1 + \beta^{1/4} + \beta^{3/4}} < \frac{2}{3}, \tag{5}$$

which means that the savings amount in the typical hyperbolic discounting model cannot exceed that in the long-term exponential economy. As shown in Laibson (1996, 1997) and Harris and Laibson (2001), with a CES preference and constant hyperbolic discounting factor, the savings amount cannot exceed that under the long-term exponential preferences. However, in the case where hyperbolic discounting factors are not constant over time (i.e., $\beta_1 \neq \beta_2$) and β_2 is sufficiently low, this example shows that the savings amount in period 1 can be even higher than that under the long-term exponential economy (i.e., $\beta_1 = \beta_2 = 1$).

Specifically, the savings in Eq. (4) increase as β_2 decreases, which means that increased the future self's myopia increases the current self's savings.[7] We interpret this effect as the commitment incentive in this paper.

For example, where $(\beta_1, \beta_2) = (0.8, 0.8)$, we have $s_1(0.8, 0.8) = 0.64$ which is lower than $s_1(1,1) = 0.67$. However, for lower values of β_2, for example $\beta_2 = 0.1$, we have $s_1(0.8, 0.1) = 0.68$, which is higher than $s_1(1,1)$: $s_1(0.8, 0.1) > s_1(1,1)$. This counter-intuitive result indicates that the savings decision under myopic discounting preferences cannot be explained only by the myopia effect. There is another effect that significantly affects savings decisions: the commitment effect, which is determined by the future self's myopia discounting factor.

3. The model

This section introduces a three-period consumption-savings decision model with hyperbolic discounting. In this model, hyperbolic discounting factors are not constant over time. However, this does not necessarily mean that the two different period selves have different degrees of myopia. This distinction is necessary to distinguish the myopic effect governed by β_1 from the self-control effect governed by β_2 in the consumer's consumption-savings decisions. In period 1, the consumer is endowed with wealth y and makes decisions on consumption and savings. In period 2, she decides on period-2 consumption and savings. In period 3, she consumes all her wealth. In a complete market, the following is the set of budget constraints:

$$c_1 + ps_1 = y \tag{6}$$

$$c_2 + s_2 = R_2 s_1 \tag{7}$$

$$c_3 = R_3 s_2 \tag{8}$$

where s_1 and s_2 are the amount of savings in periods 1 and 2, respectively, and R_2 and R_3 are the exogenously given gross interest rates in periods 2 and 3, respectively. In Eq (6), p represents the price of savings, which should be equal to 1. The reason

[7] Specifically, we have

$$\frac{\partial s_1(\beta_1, \beta_2)}{\partial \beta_2} = \frac{-\sqrt{\beta_1}(1-\sqrt{\beta_2})}{4\beta_2^{3/4}\left(\sqrt{\beta_1} + \beta_2^{1/4} + \sqrt{\beta_1\beta_2}\right)} < 0.$$

we assume the price of savings as a variable p rather than fixed at 1 is to separately quantify the impact of the myopia and self-control effects on the demand for savings, which is expressed as a relationship between s_1 and p.

We assume $u(c)$ is strictly increasing, strictly concave, twice-continuously differentiable, $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$. The lifetime utility of the consumer's self in period 1 is represented by

$$U_1(c_1, c_2, c_3) = u(c_1) + \beta_1 \delta u(c_2) + \beta_1 \delta^2 u(c_3), \tag{9}$$

where $\beta_1 \in (0, 1]$ is the myopic discounting factor for the period-1 self and $\delta \in (0, 1)$ is a long-term discounting factor.

The preference of the consumer's self in period 2 is

$$U_2(c_2, c_3) = u(c_2) + \beta_2 \delta u(c_3), \tag{10}$$

where $\beta_2 \in (0, 1]$ is the myopic discounting factor for the period-2 self. Where $\beta_1 = \beta_2$, this model is the same as the typical hyperbolic discounting function.

4. Commitment and myopic values

In this section, we show how commitment and myopia values are attributed in the demand price of savings. We assume that the consumer is sophisticated, and thus she foresees that she will have self-control problems in the future. Thus, the consumer's decision represents a subgame-perfect equilibrium, which can be derived by backward induction. Accordingly, in the second period, the sophisticated consumer chooses s_2, given s_1, to maximize U_2. Therefore, s_2 can be written as a function of s_1, that is, $s_2(s_1)$. Substituting the period-2 savings function $s_2(s_1)$ into the period-1 maximization problem, we can solve for the period-1 savings (s_1). Specifically, in period 2, the maximization problem is

$$\max_{s_2} u(R_2 s_1 - s_2) + \beta_2 \delta u(R_3 s_2), \tag{11}$$

and the first-order condition is

$$-u'(R_2 s_1 - s_2) + \beta_2 \delta u'(R_3 s_2) R_3 = 0. \tag{12}$$

From Eq. (12), we can derive the period-2 savings in terms of s_1, that is, $s_2(s_1)$. Plugging $s_2(s_1)$ into the first-period maximization problem, we have

$$\max_{s_1} u(y - ps_1) + \beta_1 \delta u(R_2 s_1 - s_2(s_1)) + \beta_1 \delta^2 u(R_3 s_2(s_1)), \tag{13}$$

whose first-order condition is

$$-pu'(c_1) + \beta_1 \delta u'(c_2)\{R_2 - s_2'(s_1)\} + \beta_1 \delta^2 u'(c_3) R_3 s_2'(s_1) = 0, \tag{14}$$

which is, in turn, equivalent to

$$pu'(c_1) = R_2 \beta_1 \delta u'(c_2) - \beta_1 \delta u'(c_2) s_2'(s_1) + R_3 \beta_1 \delta^2 u'(c_3) s_2'(s_1) \tag{15}$$

where

$$s_2'(s_1) = \frac{u''(c_2) + \beta_2 \delta R_3^2 u''(c_3)}{u''(c_2) R_2} > 0$$

From Eqs. (12) and (15), we have

$$pu'(c_1) = R_2 \beta_1 \delta u'(c_2) + s_2'(s_1) u'(c_3) R_3 \delta^2 \beta_1 (1 - \beta_2)$$

From Eq. (**??**), we have

$$p = \underbrace{\beta_1 \delta R_2 \frac{u'(c_2)}{u'(c_1)}}_{\text{Myopia value:}M(s_1)} + \underbrace{\beta_1 \delta^2 (1 - \beta_2) R_3 s_2'(s_1) \frac{u'(c_3)}{u'(c_1)}}_{\text{Commitment value:}C(s_1)} \tag{16}$$

In Eq. (16), the value of savings (p) is decomposed into two terms. The first term is the myopia effect, which is directly controlled by the myopic parameter (β_1). The second term is what we refer to as the commitment effect. The savings (s_1) can be a commitment device in the sense that the current self is willing to pay for purchasing liquid assets more than the marginal benefit (that is the myopia value, $M(s_1)$) to affect the future self's budget constraint. The impact of β_1 and β_2 on the commitment incentive would be the opposite.

The commitment effect is increasing in β_1 but decreasing in β_2. As β_1 decreases (i.e., higher myopia), the consumer has a weaker incentive to value future consumptions, so the commitment incentive for savings decreases. However, as β_2 decreases (i.e., having more serious self-control problem), the consumer has a stronger incen-

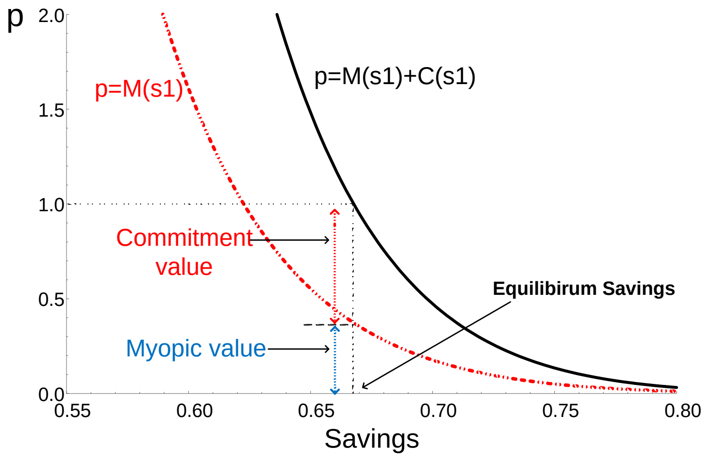

Figure 1: Myopic and commitment values in the inverse savings demand function in the example of $y = 1; \beta_1 = 0.8; \beta_2 = 0.2; R_2 = 1; R_3 = 1; \varepsilon = 0.2; \delta = 1$.

tive to overcome the future under-consumption problem by saving more. In equation (16), if $\beta_2 = 1$ (i.e., if there is no incentive for consumer 1 to make a commitment), the equation is the same as the typical Euler equation defined by the intertemporal discounting factor $\beta_1\delta$. From Eq. (16), we have the following proposition:

Proposition 1 *The commitment value is positive if and only if $\beta_2 < 1$. If $\beta_2 = 1$, a commitment value does not exist, so the Euler equation in Eq. (16) is the same as the typical time-consistent Euler equation with discounting factor $\beta_1\delta$.*

Proposition 1 indicates that the consumer's savings decisions are determined by the myopia effect and self-control effect if $\beta_2 < 1$. To summarize, (1) a lower myopia parameter (β_1) has a negative impact on the demand for savings through both myopia and commitment effects; and (2) a lower self-control parameter (β_2) has a positive impact on the demand for savings through the commitment effect.

From Eq. (16), we can define the savings demand function as a relationship between p and s_1. Figure 1 plots the inverse savings demand curve for the example of $y = 1; \beta_1 = 0.8; \beta_2 = 0.2; R_2 = 1; R_3 = 1; \varepsilon = 0.2; \delta = 1$, where ε represents the constant intertemporal elasticity of substitution. In this example, we can derive c_1, c_2 and c_3 as functions of s_1 and substitute them into Eq. (16) to obtain the savings demand function. The lower demand function in Figure 1 is from the myopic demand, that is, $p = \beta_1 \delta R_2 u'(c_2)/u'(c_1)$ and the higher demand function is directly from Eq. (16).[8]

This section shows that consumers' saving decisions are affected by both the myopia effect and the commitment effect. However, the result does not show the relative magnitudes of these two effects. Because c_1, c_2 and c_2 in Eq. (16) are affected by β_2, we do not know how myopia and self-control are separately impacted by β_2. The only information we have from Eq. (16) is that where $\beta_2 = 1$, there is no self-control effect. Therefore, in the following section, with constant IES utility functions, we analytically investigate how the two channels are affected by β_2.

5. Constant intertemporal elasticity of substitution

In the analysis of the impact of hyperbolic discounting on savings and welfare, the constant IES preference has been in a canonical form. In this section, we use constant IES utility functions and analyze how myopia and commitment values are separately affected by the consumer's self-control problem. This analysis also broadens our understanding of how the elasticity of substitution and the degree of naivety affect consumption-savings decisions. Specifically, we assume that

$$u(c) = \begin{cases} \frac{c^{1-1/\varepsilon}-1}{1-1/\varepsilon} & \text{if} \quad \varepsilon \neq 1, \\ \ln c & \text{if} \quad \varepsilon = 1. \end{cases} \tag{17}$$

[8] Specifically, in the equation, we have $c_1 = 1 - s_1, c_2 = (R_2 - a)s_1, c_3 = R_3 a s_1$ so the inverse demand curves are from

$$\text{Myopia value} = \beta_1 \delta R_2 \left(\frac{1 - s_1}{(R_2 - a)s_1} \right)^{1/\varepsilon},$$

$$\text{Commitment value} = \delta^2 \beta_1 (1 - \beta_2) R_3 a \left(\frac{1 - s_1}{R_3 a s_1} \right)^{1/\varepsilon},$$

where

$$a = R_2 (\beta_2 \delta R_3)^\varepsilon / (R_3 + (\beta_2 \delta R_3)^\varepsilon).$$

where $\varepsilon > 0$ represents the intertemporal elasticity of substitution.[9] Under constant IES preferences, Eq. (16) can be expressed in terms of s_1 and β_2:

$$p = \underbrace{\beta_1 \delta R_2 \left(\frac{1 - s_1}{(R_2 - a(\beta_2)) s_1} \right)^{1/\varepsilon}}_{\text{Myopia effect}} + \underbrace{\delta^2 \beta_1 (1 - \beta_2) R_3 a(\beta_2) \left(\frac{1 - s_1}{R_3 a(\beta_2) s_1} \right)^{1/\varepsilon}}_{\text{Commitment effect}} \tag{18}$$

where

$$a(\beta_2) = \frac{R_2 (\beta_2 \delta R_3)^{\varepsilon}}{R_3 + (\beta_2 \delta R_3)^{\varepsilon}}.$$

For the mathematical derivation of Eq. (18), see the Appendix. The right-hand side of Eq. (18) is a function of s_1, in which c_2 and c_3 do not appear. Therefore, Eq. (18) is interpreted as an inverse savings-demand function, representing a relationship between p and s_1. The following proposition shows how the myopia and commitment values are attributed in the savings demand as β_2 is changing.

Proposition 2 *With constant IES preferences, where $\varepsilon \leq 1/(1-\beta_2)$, as β_2 decreases (i.e., the consumer faces a more serious self-control problem), the myopia value decreases and the commitment value increases. That is, for any value of $s_1 \in (0, y)$, we have*

$$\frac{\partial M(\beta_2; s_1)}{\partial \beta_2} > 0 \tag{19}$$

$$\textit{and } \frac{\partial C(\beta_2; s_1)}{\partial \beta_2} < 0, \tag{20}$$

where

$$M(\beta_2; s_1) = \beta_1 \delta R_2 \left(\frac{1 - s_1}{(R_2 - a(\beta_2)) s_1} \right)^{1/\varepsilon}$$
$$\textit{and } C(\beta_2; s_1) = \delta^2 \beta_1 (1 - \beta_2) R_3 a(\beta_2) \left(\frac{1 - s_1}{R_3 a(\beta_2) s_1} \right)^{1/\varepsilon}.$$

Proposition 2 implies that as β_2 decreases, the increased commitment value generally makes the inverse saving function shift to the right while the decreased myopic value make it shift to the left. The condition $\varepsilon \leq 1/(1-\beta_2)$ indicates that the results

[9] If ε is greater than one, the limit conditions (i.e., $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$) in Section 3 would be violated. However, the conditions are necessary to ensure interior solutions. As long as interior solutions exist, all the results in this paper hold without limit conditions. Therefore, in this section, we assume that equilibrium exists as a form of interior solutions.

hold if the IES is smaller than one or if β_2 is close to one. Given that most empirical studies indicate that the value of IES is less than one, the result in Proposition 2 implies that the commitment value actually increases with a stronger self-control problem.[10]

As indicated in the previous section, the commitment value in Eq. (20) increases with decreased β_2. However, the impact of β_2 on the myopic value is the opposite to that on the commitment value, as shown in Eq. (19). As β_2 decreases, the period-2 self would be more myopic and thus period-2 consumption would be high. This high amount of period-2 consumption decreases the marginal value of period-1 savings, so the myopic value decreases.

Because the direction of the two effects is the opposite, we do not know whether a stronger self-control problem increases or decreases period-1 savings. In this paper, we show that elasticity of substitution plays an important role in determining the magnitude of the myopia and self-control effect.

Proposition 3 *For any value of* $s_1 \in (0, y)$*, we have*

$$\frac{\partial M(\beta_2; s_1)}{\partial \beta_2} + \frac{\partial C(\beta_2; s_1)}{\partial \beta_2} > 0 \textit{(respectively, =0, <0)}$$

if $\varepsilon < 1$ *(respectively, =1, >1)*

Proposition 3 indicates that the savings inverse demand function shifts to the right (the left) as β_2 decreases if the IES is smaller (larger) than one. For a lower IES, the period-1 self has a strong incentive to commit to future consumption. Therefore, for the lower elasticity of substitution, as β_2 decreases, the commitment incentive increases substantially.

Proposition 3 also shows that the decreased β_2 can increase savings only if the IES is lower than one. In an extreme case where the IES is zero, the myopia effect converges to zero and only the commitment effect remains. Since empirical results in both micro and macroeconomics suggest that the IES is lower than 1,[11] the result

[10] When the IES is very high and β_2 is very low, the commitment value can decrease with decreased β_2. If β_2 is very low and the utility is highly substitution-oriented, the period-2 self will not effectively increase period-2 savings through higher period-1 savings, so the commitment value is not decreasing in β_2. One example of this case is that $\varepsilon = 10, \beta_1 = 0.1, \delta = 1, R_3 = 3$. We can also consider the case of perfect substitution. If β_2 is very low, the period-2 self would not save at all (because period-3 consumption does not contribute intertemporal utility) thus there would be no commitment incentive.

[11] See Hall (1988), Eichenbaum and Hansen (1990), and Ogaki and Reinhart (1998).

shows that as the consumer has a more serious self-control problem, the commitment incentive for savings is more dominant than the myopia effect.

6. Naive and sophisticated choices

It is well known that with log-linear preferences, period-1 savings levels for the naive and sophisticated consumers are the same (see Pollak, 1968 and Selden and Wei, 2016). Naive or partially naive people do not fully realize their self-control problems. O'Donoghue and Rabin (2001) specifically model naive choices as assuming the naive person believes that her future hyperbolic discounting parameter is $\widetilde{\beta}_2$, which is larger than the future self's actual hyperbolic parameter β_2. If $\widetilde{\beta}_2 = 1$ (respectively, $\widetilde{\beta}_2 \in (\beta_2, 1)$, $\widetilde{\beta}_2 = \beta_2$), the consumer behaves fully naively (respectively, partially naively, and sophisticatedly). The naive person's maximization problem in Eqs. (11) and (13) is defined with $\widetilde{\beta}_2$ instead of β_2, so the Euler equation of Eq. (16) can be replaced with

$$p = \underbrace{\beta_1 \delta \frac{R_2 u'(\widetilde{c}_2)}{u'(c_1)}}_{\text{Myopic value}} + \underbrace{\delta^2 \beta_1 \left(1 - \widetilde{\beta}_2\right) \frac{\widetilde{s}_2'(s_1) R_3 u'(\widetilde{c}_3)}{u'(c_1)}}_{\text{Commitment value}}, \tag{21}$$

where $\widetilde{c}_2$, $\widetilde{c}_3$ and $\widetilde{s}_2'(s_1)$ are the period-1 self's misestimated consumption and savings based on the consumer's erroneous belief under $\widetilde{\beta}_2$. Because Eq. (21) is the same as Eq. (16) if $\widetilde{\beta}_2 = \beta_2$, the result in Proposition 3 can be directly applied to this case. Proposition 3 shows that as β_2 decreases, the demand for savings increases if the IES is lower than one. This implies that the sophisticated savings demand (with β_2) is higher than the naive consumer's savings demand (with $\widetilde{\beta}_2$ that is higher than β_2). Therefore, we can infer that the sophisticated consumer (with β_2) saves more than the naive consumer (with $\widetilde{\beta}_2$) if the IES is lower than one because we have $\beta_2 < \widetilde{\beta}_2$.

Proposition 3 also implies that if the IES is one, the naive and sophisticated period-1 savings decisions always agree. The fully naive consumer does not realize need for commitment, so the commitment value is zero. However, the naive consumer's myopia value is higher than the sophisticated consumer. This is because the naive consumer underestimates her period-2 consumption as she believes that her future self will save more than she actually does. The underestimation of period-2 consumption makes the marginal value of savings to be high in Eq. (21), so the myopia value is high.

If IES is one, the opposing effects offset each other, so the naive and sophisticated choices agree.

These findings about the IES and the degree of naivety are not new in this paper since they have been shown in many previous works (See Pollak 1968, Salanié and Treich 2006, Selden and Wei 2016). The new finding in this paper shows how the commitment effect has a role in determining the equilibrium savings of sophisticated and naive decisions for different levels of IES. From the result in this section, we can conclude that the commitment incentive for liquid savings increases as the consumer becomes more sophisticated and the IES becomes lower.

7. Steady-state analysis

In this section, we investigate how much of the commitment value is attributed to the value of the liquid financial asset using Laibson's (1997) steady state approach. The representative consumer's budget constraint in year t is

$$c_t + k_{t+1} = R_t k_t + h_t, \tag{22}$$

where h_t is labor income and k_t is the capital amount in period t. The same as in Laibson (1997), the budget constraint in (22) naturally implies that capital is liquid. By the dynamic budget constraint and the capital market clearing conditions, in equilibrium we have

$$R_t k_t + h_t = f_t(k_t) + (1-d)\, k_t, \tag{23}$$

$$R_t = f_t'(k_t) + (1-d), \tag{24}$$

and

$$h_t = f_t(k_t) - k_t f_t'(k_t), \tag{25}$$

where $f_t(k_t)$ is the per-capita production function in year t, and d is the capital depreciation rate.

We assume that the hyperbolic discounting factor for the current self is different from the future selves. Similar to the three-period model in this paper, the current self in period t's hyperbolic discounting factor is β_1 , while all future selves' factor is β_2. This distinction between β_1 and β_2 is not to assume that $\beta_1 \neq \beta_2$, but to compute the myopia and commitment effects separately. In this section, we assume that $\beta_1 =$

$\beta_2 = 0.7$, following Angeletos et al. (2001) and Laibson, Repetto, and Tobacman (2007).

We assume that the representative consumer lives T periods, which can be extended to infinite periods (i.e., $T \to \infty$) in the steady-state analysis. Assuming that period t is the current period, the self t's intertemporal utility under quasi-hyperbolic time preferences is

$$U^{(t)}(c_t, c_{t+1}, \ldots, c_T) = u(c_t) + \beta_1 \sum_{i=1}^{T-t} \delta^i u(c_{t+i}), \tag{26}$$

and the intertemporal utilities of self $t+i$ for $i = 1, 2, 3, \ldots, T$ are

$$U^{(t+i)}(c_{t+i}, c_{t+i+1}, \ldots, c_T) = u(c_{t+i}) + \beta_2 \sum_{j=1}^{T-t-i} \delta^i u(c_{t+i+j}),$$

where $u(c)$ is constant IES utility functions:

$$u(c) = \begin{cases} \frac{c^{1-\rho}-1}{1-\rho} & \text{if} \quad \rho > 1, \\ \ln c & \text{if} \quad \rho = 1. \end{cases}$$

where ρ is the inverse value of IES. In this economy, we can derive the following Euler equation:

Proposition 4 *In the hyperbolic economy with β_1 and β_2, the Euler equation is characterized as*

$$u'(c_t) = \beta_1 R_{t+1} \delta u'(c_{t+1}) + \left(\frac{1-\beta_2}{\beta_2} \right) \beta_1 R_{t+1} \delta u'(c_{t+1}) \left(1 - \Lambda_{t+1}\right), \tag{27}$$

where $\Lambda_{t+1} \in (0, 1)$ represents the marginal consumption with respect to wealth in period t.

Eq. (27) in Proposition 4 is the Euler equation for the multiple-period model with constant IES preferences and heterogenous hyperbolic factors. In Proposition 4, the value of savings (capital holding) is divided into the myopia and the commitment values. If there is no self-control problem (i.e., $\beta_2 = 1$), Eq. (27) is the same as the typical time-consistent Euler equation.

Next, we need to derive the steady state level of the gross interest rates (R_{t+1}) and the marginal consumption with respect to wealth (Λ_{t+1}) to quantitatively analyze the impact of commitment incentives. Laibson (1996) (see *Eq.* (8) at Page 11) shows that with constant IES utility functions, consumption in each period can be characterized by the following rule:

$$c_t = \lambda_{T-t} W_t, \tag{28}$$

where W_t is the sum of the financial asset and the discounted value of future labor income. From *Eq.* (27) and (28), we have the sequence $\{\lambda_i\}_{i=0}^{T}$, given by the recursion

$$\lambda_{i+1} = \frac{\lambda_i}{\left(\delta R_{i+1}^{1-\rho} \left(\lambda_i \left(\beta - 1\right) + 1\right)^{1/\rho}\right) + \lambda_i} \quad \text{and} \quad \lambda_0 = 1. \tag{29}$$

where $\beta = \beta_1 = \beta_2$. In *Eq.* (29), we can assume that R is the real interest rate under the assumption that the consumer faces the same interest rate in each period $R_1 = R_2 = \ldots = R_T$. This assumption can be used in the steady-state analysis when T is arbitrarily large ($T \to \infty$). Under this assumption, c_t converges to $\lambda^* W$, where $\lambda^* (= \lambda_i = \lambda_{i+1})$ can be derived from *Eq.* (29):

$$\lambda^* = 1 - \delta^{\frac{1}{\rho}} R^{\frac{1-\rho}{\rho}} \left[\lambda^* \left(\beta - 1\right) + 1\right]^{1/\rho}. \tag{30}$$

We consider a standard Cobb-Douglas production function, where K_t, N_t, and A_t represents aggregate capital, aggregate labor, and exogenous productivity, respectively:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{31}$$

where $k_t = K_t/N_t$. Labor is assumed to be supplied inelastically, so without loss of generality, we assume that $N_t = 1$ so that $f_t(k_t) = A_t k_t^{\alpha}$. A_t is assumed to grow exogenously at the rate g_A. Therefore, in the steady state, capital and output must grow at rate $g_A/(1-\alpha) \equiv g$. With proportional consumption, the steady state condition is

$$R(1 - \lambda^*) = \exp(g). \tag{32}$$

From *Eqs.* (30) and (32), we have the following steady-state equilibrium:

$$\exp(\rho g) = \beta \delta R + (1 - \beta)\, \delta \exp(g). \tag{33}$$

From the Cobb-Douglas production function of *Eq.* (31), the real gross interest rate R in the steady state is expressed as

$$R = \alpha \frac{Y_t}{K_t} - d + 1. \tag{34}$$

For a lower value of β, R should be higher from *Eq.* (33) and the capital-output ratio should be lower from *Eq.* (34). As Laibson (1997) showed, with $\alpha = 0.36, d = 0.08, g = 0.02$, $\beta = 1$, and $K/Y = 3$, *Eq.* (33) becomes

$$\exp(\rho(0.02)) = \delta(1.04). \tag{35}$$

Substituting *Eq.* (35) into *Eq.* (33), we have

$$1.04 = \beta R + (1 - \beta)\, e^{0.02}. \tag{36}$$

which is independent of ρ and δ. From *Eq.* (36), the steady state gross interest rate is $R = 1.04849$ if $\beta = 0.7$. Plugging $R = 1.04849$ into Eq. (34), we can obtain the steady-state capital-to-output ratio:

$$\frac{K_t}{Y_t} = \frac{\alpha}{R - (1 - d)} = 2.8018. \tag{37}$$

Because we have $Y_t = A_t K_t^{\alpha}$, from Eq. (37) we can obtain the steady-state capital and output levels:

$$K_t = \left(A_t \frac{\alpha}{R - (1 - d)} \right)^{1/(1-\alpha)} = 5.0017 A_t^{1.5625}, \tag{38}$$

$$Y_t = 1.7852 A_t^{1.5625}. \tag{39}$$

Substituting Eqs. (38), (39) and $\lambda^*(= \Lambda_{t+1})$ into Eq. (27), at the steady-state we have

$$p = \underbrace{\beta R \delta \frac{u'(c_{t+1})}{u'(c_t)}}_{\text{Myopic value } = M(K_{t+1})} + \underbrace{(1 - \beta)\, R\delta\, (1 - \lambda^*)\, \frac{u'(c_{t+1})}{u'(c_t)}}_{\text{Commitment value } = C(K_{t+1})}. \tag{40}$$

where

$$c_t = 1.785\,2 A_t^{1.5625} + (1 - d) 5.0017 A_t^{1.5625} - K_{t+1}, \tag{41}$$

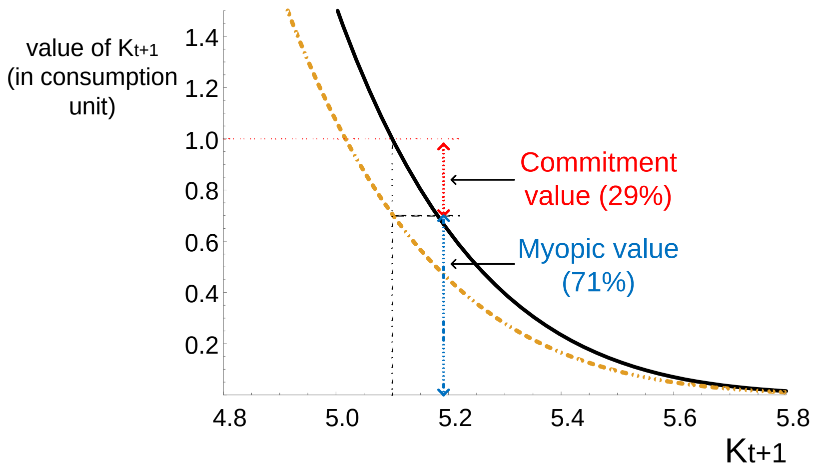

Figure 2: Myopic and commitment values in the choice of K_{t+1} in a steady state where $A_t = e^{t-1}$.

$$c_{t+1} = RK_{t+1} + h_{t+1} - e^{0.02}K_{t+1} \quad \text{and} \quad h_{t+1} = A_{t+1}K_{t+1}^{\alpha}/(1-\alpha) \tag{42}$$

Plotting the savings (capital-holding) demand function of Eq. (40) with $A_t = e^{t-1}$, we have the following demand curve described in Figure 2. The solid curve represents the inverse capital demand function, $p = M(K_{t+1}) + C(K_{t+1})$, and the dashed curve represents the demand from myopia effect, $p = M(K_{t+1})$. This numerical analysis from Eqs. (40-42) shows that in the steady state, 29% of the value of savings originates from the commitment incentive.[12]

8. Conclusion

This paper shows that under myopic preferences, the consumer has a commitment incentive to increase savings for the purpose of commitment. In the leading example, we show that this incentive can significantly increase savings, which can be even larger than the savings under exponential discounting with $\beta = 1$. This finding is quite different from our conventional understanding of hyperbolic discounting. Previous research in this area has focused on the pure myopia effect of a hyperbolic discounting

[12] In this steady-state analysis, the commitment value is not affected by the elasticity-of-substitution since the calibration is designed such that the capital-to-output ratio is invariant to the elasticity-of-substitution. Specifically, as the elasticity-of-substitution decreases, the long-term discounting factor δ increases to make the savings amount constant (See Eq. (35)).

factor. Our finding is that the current hyperbolic discounting factor can be a measure of myopia, but the future self's hyperbolic discounting factor should be interpreted as future self-control problems. Therefore, as the future self's hyperbolic discounting factor decreases, the consumer can increase current savings through commitment incentives. Using Laibson's steady-state analysis, we show that this commitment incentive significantly affects consumption-savings decisions.

To effectively show the distinct effects of myopia and self-control, we assume that the hyperbolic discounting factors can change over time. Although this paper is agnostic to the idea of changing self-control at consumer ages, there are empirical evidence (See Ameriks et al., 2007) and implication for tax policies under this assumption (See Pavoni and Yazici, 2017 and Kang and Ye, 2019). The results in this paper show that when self-control changes over time, the consumer's saving decision can be distinct from our previous understanding about the hyperbolic economy. Thus, further studies on tax polices and other applications could result in very distinct outcomes under the assumption of changing self-control.

Appendices

A. Proof of Proposition 1

In Eq. (16), if $\beta_2 \neq 1$, the commitment value is positive since we have $s_2'(s_1) > 0$. If $\beta_2 = 1$, Eq. (16) becomes $p = \beta_1 \delta R_2 u'(\widetilde{c}_2)/u'(c_1)$, which is equivalent to the typical Euler equation with discounting factor $\beta_1 \delta$:

$$pu'(c_1) = \beta_1 \delta R_2 u'(\widetilde{c}_2).$$

B. Proof of Proposition 2

With constant IES preferences, the myopia value is computed as

$$
\begin{aligned}
M(s_1;\beta_2) &= \beta_1\delta\frac{R_2u'(c_2)}{u'(c_1)} = \beta_1\delta R_2\left(\frac{c_1}{c_2}\right)^{1/\varepsilon} \\
&= \beta_1\delta\frac{R_2}{(R_2-a(\beta_2))^{1/\varepsilon}}\left(\frac{1-s_1}{s_1}\right)^{1/\varepsilon}. \qquad (43)
\end{aligned}
$$

where

$$
a(\beta_2) = \frac{R_2(\beta_2\delta R_3)^{\varepsilon}}{R_3+(\beta_2\delta R_3)^{\varepsilon}}.
$$

The commitment value is computed as

$$
\begin{aligned}
C(s_1;\beta_2) &= \delta^2\beta_1(1-\beta_2)\frac{s_2'(s_1)R_3u'(c_3)}{u'(c_1)} \\
&= \beta_1\delta^2(1-\beta_2)(R_3a(\beta_2))^{1-1/\varepsilon}\left(\frac{1-s_1}{s_1}\right)^{1/\varepsilon} \qquad (44)
\end{aligned}
$$

With both the myopia and commitment effect, there is a common term $\left(\frac{1-s_1}{s_1}\right)^{1/\varepsilon}$. Thus, to understand how β_2 affects the equilibrium savings, we need to compare how $\beta_1\delta R_2(R_2-a(\beta_2))^{-1/\varepsilon}$ and $\beta_1\delta^2(1-\beta_2)(R_3a(\beta_2))^{1-1/\varepsilon}$ in Eqs. (43) and (44) are affected by β_2. Differentiating $M(s_1;\beta_2)$ and $C(s_1;\beta_2)$ with respect to β_2, we have

$$
\frac{\partial M(s_1;\beta_2)}{\partial\beta_2}/\left(\frac{1-s_1}{s_1}\right)^{1/\varepsilon} = \frac{\beta_1\delta}{\beta_2R_3}t(\beta_2)\left(\frac{R_2R_3}{R_3+t(\beta_2)}\right)^{1-\frac{1}{\varepsilon}} > 0, \qquad (45)
$$

and

$$
\begin{aligned}
&\frac{\partial C(s_1;\beta_2)}{\partial\beta_2}/\left(\frac{1-s_1}{s_1}\right)^{1/\varepsilon} \\
&= -\delta^2\beta_1(R_3a(\beta_2))^{1-1/\varepsilon} + \left(1-\frac{1}{\varepsilon}\right)\delta^2\beta_1(1-\beta_2)(R_3a(\beta_2))^{1-1/\varepsilon}R_3\frac{\partial a(\beta_2)}{\partial\beta_2} \\
&= -\frac{\beta_1\delta^2}{\beta_2}\left[\frac{(R_2t(\beta_2))^{1-1/\varepsilon}((1+(\beta_2-1)\varepsilon)R_3+\beta_2t(\beta_2))}{(R_3+t)^{1/\varepsilon}}\right]. \qquad (46)
\end{aligned}
$$

where

$$
t(\beta_2) = (\beta_2\delta R_3)^{\varepsilon}.
$$

Eq. (45) shows that the myopia value increases in β_2 for given values of $(\varepsilon, \beta_1, \beta_2, \delta, R_2, R_3)$. Eq. (46) indicates that the sufficient condition for the commitment value to decrease in β_2 is $(1 + (\beta_2 - 1)\varepsilon) > 0$.

C. Proof of Proposition 3

From the proof of Proposition 2, we can define the function $V(\varepsilon)$ from $M(s_1; \beta_2)$ and $C(s_1; \beta_2)$ as

$$
\begin{aligned}
V(\varepsilon) &= -\frac{\partial M(s_1;\beta_2)/\partial \beta_2}{\partial C(s_1;\beta_2)/\partial \beta_2} = \frac{R_3^{1/\varepsilon}\left(\left(1-\varepsilon\right)R_3 + \beta_2 R_3 \varepsilon + \beta_2\left(\beta_2 \delta R_3\right)^{\varepsilon}\right)}{R_3 \beta_2\left(R_3 + \left(\beta_2 \delta R_3\right)^{\varepsilon}\right)} \qquad (47) \\
&= \frac{R_3^{1/\varepsilon}\left(\left(1-\varepsilon\right)R_3 + \beta_2\left(R_3 \varepsilon + \left(\beta_2 \delta R_3\right)^{\varepsilon}\right)\right)}{R_3 \beta_2\left(R_3 + \left(\beta_2 \delta R_3\right)^{\varepsilon}\right)},
\end{aligned}
$$

From Eq. (47), we know that $V(\varepsilon)$ is 1 if $\varepsilon = 1$. Differentiating $V(\varepsilon)$ with respect to ε , we have

$$
\begin{aligned}
V'(\varepsilon) = &-\frac{R_3^{-1+1/\varepsilon}}{\beta_2 \varepsilon^2 (R_3 + (\beta_2 \delta R_3)^{\varepsilon})^2} \times \\
&\{(R_3 + (\beta_2 \delta R_3)^{\varepsilon})((1 - (1 - \beta_2)\varepsilon) R_3 + \beta_2\left(\beta_2 \delta R_3\right)^{\varepsilon}) \log(R_3) \\
&(1 - \beta_2)\varepsilon^2 R_3\left(R_3 + \left(\beta_2 \delta R_3\right)^{\varepsilon} + \left(1 - \varepsilon\right)\left(\beta_2 \delta R_3\right)^{\varepsilon} \log(\beta_2 \delta R_3)\right)\},
\end{aligned}
$$

which is zero (strictly negative) if $\varepsilon = 1$ (> 1). This implies that for any value of $s_1 \in (0, y)$, we have

$$
\frac{\partial M(\beta_2; s_1)}{\partial \beta_2} + \frac{\partial C(\beta_2; s_1)}{\partial \beta_2} > 0 \text{ (resp, =0, <0)}
$$

if $\varepsilon < 1$ (resp, =1, >1)

D. Proof of Proposition 4

We adopt Laibson's (1996) "partial equilibrium" approach to solve the steady state equilibrium. There are T periods. In each period, the consumer makes consumption-savings decisions. Self t chooses consumption for period t as

$$0 < c_t < W_t,$$

and, then, self $t+1$'s wealth is

$$W_{t+1} = R(W_t - c_t)$$

where R is the gross real return on wealth, which is constant over time. Wealth W_t represents the sum of current financial wealth and the present value of the stream of labor income, as Laibson (1996 and 1997) defined. With the tax policy model, W_t also includes the present value of the stream of lump-sum subsidies and taxes. This finite-period partial equilibrium model can be connected to the infinite-period steady-state model.

The budget constraint in period t is

$$c_t + k_{t+1} = R_t k_t + h_t \tag{48}$$

where h_t is the labor income. In the partial equilibrium model, h_t is considered a constant. The marginal benefit of postponing Δ units of consumption generates a stream of utility perturbations from the perspective of self t. At time t, the utility value of

$$\Delta u'(c_t) \tag{49}$$

is lost. At time $t+1$

$$\beta_1 \delta \frac{\partial c_{t+1}}{\partial W_{t+1}} R \Delta u'(c_{t+1}) \tag{50}$$

utilities are gained. Note that $\frac{\partial c_{t+j}}{\partial W_{t+j}}$ is the marginal consumption rate in period $t+j$. At time $t+2$, the utility gain is

$$\beta_1 \delta^2 \frac{\partial c_{t+2}}{\partial W_{t+2}} \left(1 - \frac{\partial c_{t+1}}{\partial W_{t+1}}\right) R^2 \Delta u'(c_{t+2}). \tag{51}$$

At time $t+i$, the utility gain is

$$\beta_1 \delta^i \frac{\partial c_{t+i}}{\partial W_{t+i}} \left[\Pi_{j=1}^{i-1} \left(1 - \frac{\partial c_{t+j}}{\partial W_{t+j}}\right)\right] R^i \Delta u'(c_{t+i}). \tag{52}$$

From *Eq.* (49) and (52), the Euler equation from the decision of period t' self is

$$u'(c_t) = \beta_1 \sum_{i=1}^{T-t} \delta^i \frac{\partial c_{t+i}}{\partial W_{t+i}} \left[\Pi_{j=1}^{i-1} \left(1 - \frac{\partial c_{t+j}}{\partial W_{t+j}} \right) \right] R^i u'(c_{t+i}). \tag{53}$$

$$= \beta_1 \delta \frac{\partial c_{t+1}}{\partial W_{t+1}} R u'(c_{t+1})$$

$$+ \left(1 - \frac{\partial c_{t+1}}{\partial W_{t+1}} \right) \beta_1 \sum_{i=1}^{T-(t+1)} \delta^{i+1} \frac{\partial c_{t+1+i}}{\partial W_{t+1+i}} \left[\Pi_{j=1}^{i-1} \left(1 - \frac{\partial c_{t+1+j}}{\partial W_{t+1+j}} \right) \right] R^{i+1} u'(c_{t+i+1}).$$

The Euler equation from the decision of the period $t+1$' self is

$$u'(c_{t+1}) = \beta_2 \sum_{i=1}^{T-(t+1)} \delta^i \frac{\partial c_{t+1+i}}{\partial W_{t+1+i}} \left[\Pi_{j=1}^{i-1} \left(1 - \frac{\partial c_{t+1+j}}{\partial W_{t+1+j}} \right) \right] R^i u'(c_{t+1+i}). \tag{54}$$

From *Eq.* (53) and (54), we derive the following Euler equation on the unique equilibrium path:

$$u'(c_t) = \beta_1 \delta \frac{\partial c_{t+1}}{\partial W_{t+1}} R u'(c_{t+1}) + \left(1 - \frac{\partial c_{t+1}}{\partial W_{t+1}} \right) \frac{\beta_1 \delta R}{\beta_2} u'(c_{t+1}) \tag{55}$$

$$= R \delta u'(c_{t+1}) \left[\beta_1 \frac{\partial c_{t+1}}{\partial W_{t+1}} + \frac{\beta_1}{\beta_2} \left(1 - \frac{\partial c_{t+1}}{\partial W_{t+1}} \right) \right],$$

which is the same as Eq. (27), where $\frac{\partial c_{t+1}}{\partial W_{t+1}} = \Lambda_{t+1}$.

References

Ainslie, G. and V. Haendel (1983). The motives of the will. In T. S. E. Gottheil, K. Druley and H. Waxman (Eds.), *Etiologic Aspects of Alcohol and Drug Abuse.* Springfield, IL: Charles C. Thomas.

Amador, M., I. Werning, and G.-M. Angeletos (2006). Commitment vs. flexibility. *Econometrica 74*(2), 365–396.

Ameriks, J., A. Caplin, J. Leahy, and T. Tyler (2007). Measuring self-control problems. *American Economic Review 97*(3), 966–972.

Angeletos, G.-M., D. Laibson, A. Repetto, J. Tobacman, and S. Weinberg (2001). The hyperbolic consumption model: Calibration, simulation, and empirical evaluation. *Journal of Economic Perspectives 15*(3), 47–68.

Bénabou, R. and J. Tirole (2002). Self-confidence and personal motivation. *Quarterly Journal of Economics 117*(3), 871–915.

Bénabou, R. and J. Tirole (2004). Willpower and personal rules. *Journal of Political Economy 112*(4), 848–886.

Dekel, E., B. L. Lipman, and A. Rustichini (2001). Representing preferences with a unique subjective state space. *Econometrica 69*(4), 891–934.

Dekel, E., B. L. Lipman, and A. Rustichini (2009). Temptation-driven preferences. *Review of Economic Studies 76*(3), 937–971.

Diamond, P. and B. Köszegi (2003). Quasi-hyperbolic discounting and retirement. *Journal of Public Economics 87*(9-10), 1839–1872.

Eichenbaum, M. and L. P. Hansen (1990). Estimating models with intertemporal substitution using aggregate time series data. *Journal of Business & Economic Statistics 8*(1), 53–69.

Frederick, S., G. Loewenstein, and T. O'Donoghue (2002). Time discounting and time preference: A critical review. *Journal of Economic Literature 40*(2), 351–401.

Goldman, S. M. (1979). Intertemporally inconsistent preferences and the rate of consumption. *Econometrica 47*(3), 621–626.

Gul, F. and W. Pesendorfer (2001). Temptation and self-control. *Econometrica 69*(6), 1403–1435.

Gul, F. and W. Pesendorfer (2004a). Self-control and the theory of consumption. *Econometrica 72*(1), 119–158.

Gul, F. and W. Pesendorfer (2004b). Self-control, revealed preference and consumption choice. *Review of Economic Dynamics 7*(2), 243–264.

Hall, R. E. (1988). Intertemporal substitution in consumption. *Journal of Political Economy 96*(2), 339–357.

Harris, C. and D. Laibson (2001). Dynamic choices of hyperbolic consumers. *Econometrica 69*(4), 935–957.

Kang, M. and L. S. Ye (2019). Present bias and corporate tax policies. *Journal of Public Economic Theory 21*(2), 265–290.

Laibson, D. (1996). Hyperbolic discount functions, undersaving, and savings policy. *National bureau of economic research.*

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics 112*(2), 443–477.

Laibson, D., A. Repetto, and J. Tobacman (2007). Estimating discount functions with consumption choices over the lifecycle. Technical report, National Bureau of Economic Research.

Loewenstein, G. and D. Prelec (1992). Anomalies in intertemporal choice: Evidence and an interpretation. *Quarterly Journal of Economics 107*(2), 573–597.

O'Donoghue, T. and M. Rabin (1999). Doing it now or later. *American Economic Review 89*(1), 103–124.

O'Donoghue, T. and M. Rabin (2001). Choice and procrastination. *Quarterly Journal of Economics 116*(1), 121–160.

Ogaki, M. and C. M. Reinhart (1998). Measuring intertemporal substitution: The role of durable goods. *Journal of Political Economy 106*(5), 1078–1098.

Pavoni, N. and H. Yazici (2017). Optimal life-cycle capital taxation under self-control problems. *Economic Journal 127*(602), 1188–1216.

Phelps, E. S. and R. A. Pollak (1968). On second-best national saving and game-equilibrium growth. *Review of Economic Studies 35*(2), 185–199.

Pollak, R. A. (1968). Consistent planning. *Review of Economic Studies 35*(2), 201–208.

Salanié, F. and N. Treich (2006). Over-savings and hyperbolic discounting. *European Economic Review 50*(6), 1557–1570.

Selden, L. and X. Wei (2016). Changing tastes and effective consistency. *Economic Journal 126*(595), 1912–1946.

Strotz, R. H. (1956). Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies 23*(3), 165–180.

Thaler, R. (1981). Some empirical evidence on time inconsistency. *Review of Economic Studies 23*, 165–180.