Human-Capital Investments as a Commitment Device

Lawrence Jin
National University of Singapore

Minwook Kang
Korea University

February 23, 2023

Abstract

This paper shows that a person with self-control problem can have incentive to use human-capital investment as a commitment tool to curb future myopic human-capital decisions. This commitment incentive arises from the property of increasing productivity return of human-capital accumulation. Today's larger investment in human capital increases the return of future investments in human capital, which helps encourage the future self to invest more. While time-inconsistency has often been used to explain why people under-invest in human capital, such as education and health, our findings suggest that time-inconsistency can also explain why some people make very *large* human-capital investments.

Keywords: Human capital, hyperbolic discounting, present bias, self-control problem, commitment device, commitment incentive

JEL classifications: D02, J24, O15

This paper was presented at the 2019 Asian Meeting of the Econometric Society (Xiamen, China). We appreciate helpful comments and constructive remarks from two anonymous referees and the editor (Angus Chu). This research is supported by a Korea University Grant (K2110021). All errors are our own.

1. Introduction

Economic agents use various commitment tools to curb their future myopic behavior. The time-inconsistent hyperbolic-discounting model initially devised by Strotz (1956) and developed by Phelps and Pollak (1968), Pollak (1968) and Laibson (1997) helps explain why and how people use commitment devices.[1,2] Since then, the literature has introduced various forms of commitment devices such as illiquid financial assets (See Laibson 1997, Kocherlakota 2001), minimum savings rule (Amador et al. 2006), and information restriction (See Bénabou and Tirole 2002; 2004).[3]

Laibson (1997) notes that, "in general, all illiquid assets provide a form of commitment," and human capital is an illiquid asset that would also have commitment properties (page 445):

"...note that social security wealth and human capital, two relatively large components of illiquid wealth, are not included in the Federal Reserve Balance Sheets. Despite the abundance of commitment mechanisms, and Strotz' well-known theoretical work, intrapersonal commitment phenomena have generally received little attention from economists."

Understanding the commitment properties of human-capital investment is important because, for many people, it is the most important investment decision they make in their lifetimes. Yet to the best of our knowledge, no study has investigated the commitment properties of human-capital investments.

In this paper, we model human-capital investment decisions using hyperbolic discounting preferences, and show that human capital can provide commitment value to agents with self-control problems. The key mechanism through which human capital provides commitment is the property of increasing productivity returns to investment. That is, for human capital, today's investment positively affects the return of future human-capital investments (i.e., human capital and human-capital investments exhibit complementarity).[4] Compared to illiquid assets that have been previously studied in the literature, this is an additional channel through which

[1] Extensive introspective and empirical evidence suggest that consumers' discounting functions are approximately hyperbolic rather than exponential (Thaler 1981, Ainslie and Haendel 1992; Loewenstein and Prelec 1992; Angeletos et al. 2001).

[2] Gul and Pesendorfer (2001, 2004a, 2004b) and Dekel, Lipman, and Rustichini (2009) provided axiomatic foundations for time-inconsistent preferences where commitment devices are valued.

[3] For extensive survey work for commitment devices under present bias, see Bryan et al (2010). Recently, Bond and Sigurdsson (2018) reconsidered Amador et al (2006)'s model and provided conditions under which fully efficient commitment contracts can be devised.

[4] The property of increasing returns in *social* human capital is the key assumption in endogenous growth theory by Romer (1986, 1990). For *private* human capital accumulation, which is modeled in this paper, the empirical literature also show that it exhibits increasing returns to scales. See Saint-Paul (1996) and Acemoglu (1996).

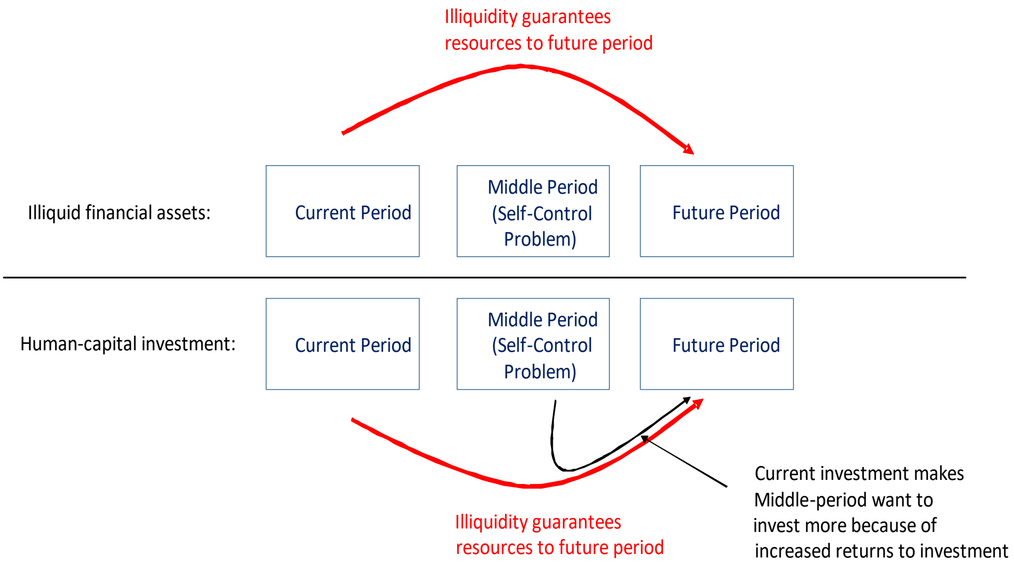

Figure 1: Illiquidity and increasing returns

today's human-capital investment affects tomorrow's behavior: not only does the illiquidity of human capital constrain tomorrow's consumption, it also affects tomorrow's incentive to invest in human capital (Figure 1). In fact, we find that illiquidity is no longer a sufficient condition for human capital to have positive commitment value.

We find that larger degree of increasing return (i.e., larger degree of complementarity between human capital and human-capital investment) generates stronger commitment value. On the other hand, if the degree of increasing return is not sufficiently large, then human capital can even provide negative commitment value. This is different from our previous understanding of the commitment properties of illiquid assets. Previously, illiquid assets could provide positive or zero, but not negative, commitment values.[5]

One implication of positive commitment value from human-capital investment is that time-inconsistent consumers would have additional incentive to invest in human capital today, to curb future myopic human-capital decisions. In fact, time-inconsistent consumers may not necessarily "under-invest" in human capital, as it has been typically understood in the literature.[6] On the one hand, time-inconsistent

[5]Laibson (1997) shows that in a complete market, the illiquid financial asset does not have commitment value.

[6]By "under-invest" we mean that time-inconsistent consumers invest less in human capital than time-consistent (exponential) consumers, where the latter is often assumed to be optimal. For example, Cadena and Keys (2015) show that impatient people more frequently drop out of educational programs, consistent with the notion that time-inconsistent consumers accrue less human capital than optimal. This is true for naive agents. In this paper, we show that time-

consumers want to invest less today (compared to a time-consistent consumer) because the reward comes later. On the other hand, time-inconsistent consumers want to invest more today because investment today would encourage the future self to invest more, which helps remedy future self-control problems. If the latter incentive is stronger than the former, the time-inconsistent consumer would invest more than a time-consistent consumer would.

We illustrate the intuition using a numerical example. A consumer lives three periods and has opportunities to invest in human capital in the first two periods. The cost of investment in the first period is \$100, and \$300 in the second period. The return of period-1 investment realized in period 2 is 30%. The investment also exhibits an increasing productivity return to future investment: the return of period-2 investment realized in period 3 is 150% in the absence of period-1 investment, and 170% in the presence of period-1 investment.

Let us assume linear utility and that both the discount rate and real net interest rate are zero. A time-consistent consumer (with δ=1) will invest in period 2 because the return in period 3 will be greater than 100%. However, she will not invest in period 1. The total return of period-1 investment is \$90, where \$30 is realized in period 2 and \$60 (\$300*(170%-150%)) is the increased return realized in period 3.

By contrast, a consumer with self-control problem may invest in period 1 to incentivize herself to invest in period 2. Consider a sophisticated quasi-hyperbolic consumer with β=0.6. She knows that if she does not invest in period 1, she will not invest in period 2 because the discounted return of 150% is less than 1 (150%$\times\beta$=0.9). However, she predicts that she will invest in period 2 if she has invested in period 1 (170%$\times\beta$>1). Her choice in period 1 is therefore between not investing in either period versus investing in both periods. The utility for investing in both periods ((\$30+\$510)$\times\beta$=\$324) is higher than never investing (\$100+\$300$\times\beta$=\$280), so she chooses to invest in period 1.[7]

While the above example assumes linear utility, we show that the curvature of the utility can greatly affect the commitment value of human capital. Human-capital accumulation increases future consumptions and thus decreases future marginal utility. If the decrease in marginal utility due to human-capital investment

inconsistent but sophisticated agents may actually accrue more human capital than time-consistent agents due to its commitment properties.

[7]In this particular example, the value of the commitment incentive is \$90. This is the sum of two parts. First, the commitment value of the investment overcomes a loss of \$46 from the \$100 investment in period 1. That is, the discounted period-1 value of the return of human-capital investment is \$54 ($\beta\times$(\$100$\times$30%+\$300*(170%-150%))), \$46 less than the cost. Second, the period-1 investment generates excess utility worth \$44 (\$324-\$280). Hence, the total value of the commitment incentive is \$90.

outweighs the increase in human-capital return, the consumer may have zero or even negative commitment incentive to invest in human capital. Specifically, this paper defines the degree of increasing return as the human-capital elasticity of investment, which measures the percentage change in human-capital return with respect to its investment. In the case where this elasticity is greater than the consumption elasticity of marginal utility, the human capital accumulation effectively induces the future self to invest more in human capital and thus can be used as a commitment tool.

If the reverse is true, human capital would induce negative commitment value. To the best of our knowledge, the possibility of a negative commitment value of illiquid assets is novel to the hyperbolic discounting literature. It is an interesting property because a negative commitment value would further deter time-inconsistent consumers from investing in human capital today, because it would *worsen* future self-control problems. An example where this may occur is education for a part-time job. If the person plans to switch to a different full-time job, then the education for the part-time job would not increase the returns of future education, and it would likely have negative commitment effects.

This paper contributes to the literature on hyperbolic-discounting model (Strotz 1956; Phelps and Pollack 1968; Pollack 1968; Laibson 1997). Previous studies have identified various commitment devices, including illiquid financial assets (Laibson 1997; Kocherlakota 2001), information restriction (Benabou and Tirole 2002; 2004), sin taxes (O'Donoghue and Rabin 2006), a minimum savings rule (Amador et al. 2006), addiction (Jin and Kang 2022), and commitment savings contracts (Bond and Sigurdsson 2018). Our paper is the first to demonstrate the commitment properties of human-capital investments. Importantly, human-capital investments differ from other illiquid assets due to the additional role increasing returns plays in determining the commitment properties of human-capital investments (Figure 1).

Our findings also have important implications for a growing number of studies investigating how hyperbolic discounting impacts individual decision-making. For example, previous studies have investigated the demand for commitment devices for addictive goods, such as alcohol and smoking (Gruber and Mullainathan 2005; Bernheim et al. 2016; Hinnosaar 2016; Schilbach 2019). The hyperbolic-discounting model has also been used to explain various decisions we make related to health (Gruber and Koszegi 2001; 2004; DellaVigna and Malmendier 2006; Ruhm 2012), energy consumption (Schleich et al. 2019; Werthschulte and Loschel 2021), participation in welfare programs and insurance (Fang and Silverman 2009; Koo and Lim 2021), as well as corporate decisions (Li et al. 2016; Kang and Ye 2019; 2021; Liu et al. 2019).

Our findings also provide new insights for the human-capital literature. Earlier studies have primarily focused on time-inconsistent preferences leading to lower human-capital investment, such as under-investment in education (Oreopoulos 2007; Cadena and Keys 2015; Levitt et al. 2016), under-investment in private health care (Newhouse 2006), and over-consumption of addictive goods (Gruber and Koszegi, 2001; 2004). These studies consider the effects of present bias, but not the commitment properties of human capital. By studying the commitment properties, we observe a novel finding that time-inconsistency can also explain why some consumers would make *larger* human-capital investments than we expect. For example, time-inconsistent consumers may want to exercise not only for the direct health benefits but also for an additional commitment benefit that helps their future selves to continue exercising later in life. Likewise, some parents may try hard to send their kids to good schools, not only for the direct benefits that good schools provide, but also for an additional benefit that it can commit the kids to continue studying harder in the future.

The remainder of this paper is organized as follows. Section 2 develops a simple three-period human capital investment model with hyperbolic discounting. Section 3 derives a demand function for human-capital investment and shows that the commitment premium has a positive value if the investment in human capital sufficiently increases future productivity return of human capital.[8] Section 4 concludes.

2. The model and the value of human-capital investment

We introduce a simple three-period human-capital investment (HCI) model with quasi-hyperbolic discounting. We define the period-utility in terms of consumption (c_t) and leisure (l_t) in period t. The period-utility at period t is given by

$$u(c_t) + v(l_t), \tag{1}$$

[8]In Appendix A, we show that the main results hold for multiplicatively-separable utility functions (such as Cobb-Douglas utility functions). Appendix B shows that the main results also hold when we incorporate endogenous labor into the model. Appendix C shows how the inclusion of consumption-savings decision affects our main result.

where u and v are continuously-differentiable, strictly increasing, and strictly concave.[9] To rule out corner solutions, we assume that $\lim_{i_t \to 0} v(l_t) = \infty$.

We assume that the individual with human capital h_t in period t is equivalent to the real income level. Thus, in this model, the consumption and human-capital levels are equivalent at each period, i.e., $c_t = h_t$ for $t = 1, 2, 3$.[10] In period 1, the consumer is endowed with human capital (h_1). The consumer is also endowed with one unit of time every period. The one unit of time is allocated into leisure (l_t) and human-capital time investment (i_t), i.e., $1 = l_t + i_t$. If the consumer allocates i_t units of time for human-capital investment in period t, the human capital stock in period $t+1$ becomes

$$h_{t+1} = H(h_t, i_t), \tag{2}$$

where H is a human-capital accumulation function. The function $H : \mathbb{R}^2_{++} \to \mathbb{R}_{++}$ is continuously-differentiable, strictly increasing, has diminishing marginal return in i_t, and for all $h_t \in \mathbb{R}_{++}$, $\lim_{i_t \to \infty} H_2(h_t, i_t) = 0$, where $H_2(h_t, i_t)$ represents the derivative of $H(h_t, i_t)$ with respect to i_t.[11]

In period 2, the consumer also makes decision on human-capital investment i_2. Then, the period utility in period 2 is $u(c_2) + v(1 - i_2)$ where $c_2 = h_2 = H(h_1, i_1)$. In period 3, which is the last period for the consumer, the period-utility is $u(c_3) + v(1)$ where $c_3 = h_3 = H(h_2, i_2)$. There is no more human-capital investment in period 3.

We incorporate this conventional model of human-capital investment into the popular β-δ quasi-hyperbolic discounting model. We assume that the consumer is sophisticated in that she knows her future preferences.[12] Therefore, we need to solve the decision process by backward induction. The period-2 self will solve the following maximization problem given the human-capital level h_2:

[9] Throughout this paper, we assume that the period utility function is additively separable. The main results also hold for multiplicatively-separable utility functions, such as Cobb-Douglas utility functions. See Appendix A for details.

[10] The simplifying assumption that $c_t = h_t$ does not harm the generalization of the results. Even if we assume that c_t is some increasing function of h_t such that $c_t = g(h_t)$, replacing the utility function $u(\cdot)$ with $u \circ g(\cdot)$, we can get the same results. In Appendix B, we show that including endogenously chosen labor hours does not change the main result of the paper, i.e., the increasing productivity return of human capital continues to play the key role in the existence of commitment incentive. To convey concisely how hyperbolic discounting triggers the commitment incentive, we keep the simple assumption in this paper.

[11] The assumption of diminishing marginal return of i_2 rules out commonly used human capital accumulation formula that is $H = h \times i$. However, this assumption is necessary for guaranteeing the existence of interior solutions for more general utility functions. In the example in this paper, we also use $H = h \times i$ and interior solution exists.

[12] If the consumer is naive, she does not realize the commitment value, thus sophistication is the natural assumption in investigating commitment incentives from human-capital investment.

$$\max_{i_2|h_2} u(h_2) + v(1 - i_2) + \beta\delta \{u(h_3) + v(1)\}. \tag{3}$$

where $\beta \in (0, 1)$ represents a hyperbolic discount factor and δ represents long-term discount factor.[13] The first-order condition from the maximization problem of (3) is

$$-v'(1 - i_2) + \beta\delta u'(h_3)H_2(h_2, i_2) = 0. \tag{4}$$

From Eq. (4), because $v(\cdot)$, $u'(\cdot)$ and $H_2(\cdot)$ are strictly monotonic functions, we have the investment i_2 as a function of h_2, that is $\widehat{i}_2(h_2) : \mathbb{R}_{++} \to [0, 1]$. Plugging the function into the period-1 maximization problem we have

$$\max_{i_1} \begin{bmatrix} u(h_1) + v(1 - i_1) \\ +\beta\delta \left\{ u(h_2) + v(1 - \widehat{i}_2(h_2)) + \delta u(h_3) + \delta v(1) \right\}. \end{bmatrix} \tag{5}$$

By the condition that $\lim_{i_t \to \infty} H_2(h_t, i_t) = 0$ and $\lim_{i_t \to 0} H_2(h_t, i_t) = \infty$, we know that the consumer does not choose either $i_1 = 0$ or $i_2 = \infty$, so there exists an interior solution i_1.

The first-order condition from the maximization problem in Eq. (5) is

$$\begin{aligned} & -v'(l_1) \\ & +\beta\delta \left\{ u'(h_2)H_2(h_1, i_1) - v'(l_2)\widehat{i_2'}(h_2)H_2(h_1, i_1) \right\} \\ & +\beta\delta^2 u'(h_3)H_1(h_2, \widehat{i}_2(h_2))H_2(h_1, i_1) \\ & +\beta\delta^2 u'(h_3)H_2(h_2, \widehat{i}_2(h_2))\widehat{i_2'}(h_2)H_2(h_1, i_1) = 0. \end{aligned} \tag{6}$$

Remembering Eq. (4), we have

$$\beta\delta u'(h_3)H_2(h_2, i_2) = v'(l_2). \tag{7}$$

From Eqs. (6) and (7), we have[14]

$$\begin{aligned} v'(l_1) &= \underbrace{\beta\delta u'(h_2)H_2(h_1, i_1) + \beta\delta^2 u'(h_3)H_1(h_2, i_2)H_2(h_1, i_1)}_{\text{Discounted Marginal Utility}} \\ & \quad + \underbrace{(1 - \beta)\, \delta\widehat{i_2'}(h_2)H_2(h_1, i_1)v'(l_2)}_{\text{Commitment value}}. \end{aligned} \tag{8}$$

[13]Regardless of the value of δ, the consumer has time-inconsistent preferences if and only if $\beta < 1$.

[14]Even with multiplicatively-separable utility functions, we have the same format of demand functions with commitment value as Eq. (8). For details, see Appendix A.

Increasing one time unit of human-capital investment in period 1, the consumer gives up utility amount equal to $v'(l_1)$. In the time-consistent model (i.e., $\beta = 1$), the utility loss of giving up one unit of time is same as the discounted future utility gain by increasing one unit of human-capital investment. However, under present-biased preferences, the utility loss is not the same as the utility gain, since there is non-zero commitment value if $\beta \neq 1$ as shown in Eq. (8).

There are two terms on the right-hand side of Eq. (8). The first term is the utility contribution of HCI and the second term is the commitment value. As the consumer increases one unit of human-capital investment in period 1, period-2 human capital is increased by $H_2(h_1, i_1)$ and period-3 human capital is increased by $H_1(h_2, i_2)H_2(h_1, i_1)$. This rise in the human-capital accumulation increases consumption in both periods 2 and 3.

With hyperbolic discounting model, there is an additional value from investing in human capital, i.e., the commitment value in Eq. (8). The following proposition shows that, under the hyperbolic discounting model, the value of human capital can be different from the utility value from increased consumption.

Proposition 1 *If $\beta < 1$ and $\widehat{i}_2'(h_2) > 0$ (resp, =0, <0), there is positive (resp, zero, negative) commitment incentive to invest in human capital.*

Proof: Directly from Eq. (8). **End of Proof.**

To visualize Proposition 1, we present a simple example in which $u(c) = \sqrt{c}$,$v(l) = l$, and $H(h, i) = h \times i$.[15] In this example, the investment choice function is[16]

$$\widehat{i}_2(h_2) = \frac{1}{2}\beta^2\delta^2 h_2 = \frac{1}{2}\beta^2\delta^2 h_1 i_1, \tag{9}$$

which is a strictly increasing function in h_2, and thus there is a positive commitment value based on Proposition 1. Let us assume that $h_1 = 1, \beta = 0.6$, and $\delta = 1$. Then, from Eqs. (8) and (9), we can get the discounted marginal utility and the commitment value as a function of i_1. In Figure 2, the upper solid curve represents the inverse demand function of HCI with shadow price of HCI (p). In this paper, we assume that investing one unit of time creates one unit of HCI, which implies the shadow price is one. However, if one unit of time creates two units of HCI, the shadow price is 0.5. Simply put, if the shadow price is p, we can get the result by

[15] The boundary conditions of $H_2(h_t, i_t)$ are necessary only for ensuring interior solutions. Even if the boundary conditions are not satisfied, as long as the interior solutions exists, the equilibrium allocations are differentiable to the value of β (as the case of the leading example).

[16] Even though this example violates the assumption of strict concavity of $v(l)$, there exists an interior solution such that all the first and second order conditions are satisfied.

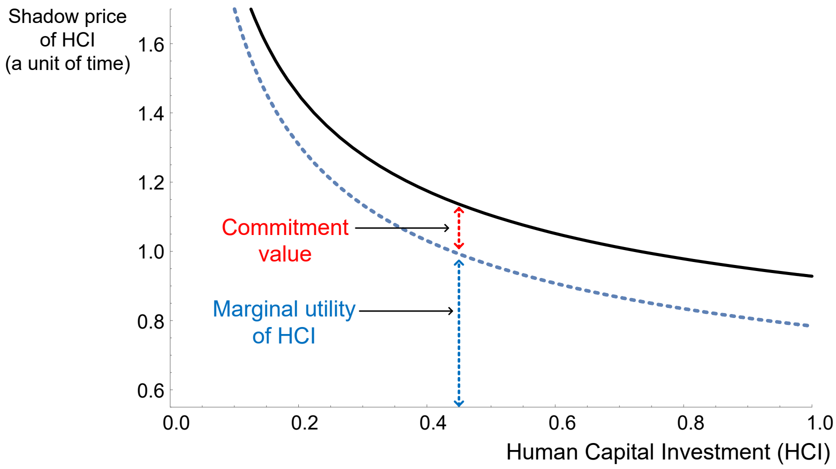

Figure 2: Commitment values in human capital investment

replacing the period-1 leisure utility with $v(1-pi_1)$ instead of $v(1-i_1)$. Specifically, Eq. (8) can be expressed, in this example, as

$$p \times \underbrace{v'(l_1)}_{=1} = \underbrace{0.849\sqrt{i_1} - 0.496 i_1}_{\text{Discounted Marginal Utility}} + \underbrace{0.144}_{\text{Commitment value}} .$$

The lower dashed curve represents discounted marginal utility. The gap between the curves represents the commitment value. Figure 2 shows the shadow price of HCI in terms of time unit decomposed into discounted marginal utility and the commitment value.[17] The figure shows that for any shadow price, there is positive commitment value in HCI.[18]

3. Increasing return and commitment incentive

In the previous section, if $\widehat{i}_2'(h_2) > 0$, there exists strictly positive commitment value in HCI. In this section, we investigate how the sign of the investment response function $\widehat{i}_2'(h_2)$ is determined by the human-capital accumulation function $H(h, i)$.

[17] If $p = 1$, the HCI level in period 1 is 0.7317 in this example.

[18] We can also provide an example where there is negative commitment value. Reversing the inequality in Eq. (12), the condition for having a negative commitment value should be $1/\varepsilon > 1/\rho$. Thus, changing the utility function as $u = -c^{-1}$ but keeping the same human capital function, we have the condition for a negative commitment value. For the same parameters in the leading example ($h_1 = 1, \beta = 0.6$, and $\delta = 1$), we have

$$p \times \underbrace{v'(l_1)}_{=1} = \underbrace{0.775 i_1^{-1} + 0.6 i_1^{-2}}_{\text{Discounted Marginal Utility}} + \underbrace{\left(-0.31 i_1^{-2}\right)}_{\text{Commitment value}} ,$$

in which the commitment value is negative.

Defining the function $f : \mathbb{R}^2_{++} \to \mathbb{R}$ such that

$$f(h_2, i_2) = u(H(h_2, i_2)),$$

the first-order condition of the period-2 maximization in Eq. (3) is

$$-v'(1 - \widehat{i}_2(h_2)) + \beta\delta f_2(h_2, \widehat{i}_2(h_2)) = 0. \tag{10}$$

Implicitly differentiating Eq. (10) with respect to h_2, we have

$$v''(l_2)\widehat{i}_2'(h_2) + \beta\delta f_{12}(h_2, i_2) + \beta\delta f_{22}(h_2, i_2)\widehat{i}_2'(h_2) = 0,$$

which is equivalent to

$$\widehat{i}_2'(h_2) = -\frac{\beta\delta f_{12}\left(h_2, \widehat{i}_2(h_2)\right)}{v''(l_2) + \beta\delta f_{22}\left(h_2, \widehat{i}_2(h_2)\right)}.$$

Because v and u are strictly concave and H is concave in i_2, we know that $v''(l_2) + \beta\delta f_{22}(h_2, i_2) < 0$. Therefore, the necessary and sufficient condition for $\widehat{i}_2'(h_2) > 0$ is $f_{12}(h_2, i_2) > 0$. Because $f_{12} = (u''H_1H_2 + u'H_{12})$, we have the following proposition:

Proposition 2 *There exists a positive commitment incentive to invest in human capital if and only if*

$$-c_3\frac{u''(c_3)}{u'(c_3)} < \frac{H \times H_{12}}{H_1 \times H_2}. \tag{11}$$

Proof: Because we have $f_{12} = (u''H_1H_2 + u'H_{12})$, the condition that $\widehat{i}_2'(h_2) > 0$ is equivalent to $u''H_1H_2 + u'H_{12} > 0$, which is in turn equivalent to the inequality of Eq. (11). **End of Proof.**

In the inequality of Eq. (11) in Proposition 2, the left side represents the curvature of the marginal utility while the right side represents the degree of complementarity between HCI and human-capital accumulation. The condition can be understood using the constant elasticity-of-substitution functions. Let us assume that the intertemporal elasticity of substitution is ε. We also assume that the elasticity of substitution between past human capital accumulation (h_2) and current human-capital investment (i_2) is ρ and $H(h, i)$ is homogeneous of degree one. Then, the condition in Eq. (11) is equivalent to

$$\frac{1}{\varepsilon} < \frac{1}{\rho}. \tag{12}$$

In Eq. (12), the inverse of the elasticity of substitution $(1/\rho)$ represents the degree of complementarity, which measures how the increase in one argument (human-capital investment) effectively increases the marginal return of the other argument (previous human-capital accumulation). On the other hand, increased future consumption due to the human-capital investment decreases the marginal utility. Thus, as the degree of complementarity of intertemporal utility $(1/\varepsilon)$ in Eq. (12) goes down, the consumer has a stronger incentive to use the human-capital investment as a commitment tool. Therefore, the main conclusion in Proposition 2 is that for HCI to be used as a commitment device, (1) HCI needs to exhibit sufficiently large increasing productivity return in past human-capital accumulation (i.e., sufficiently large degree of complementarity between human capital and human-capital investment) and (2) the intertemporal elasticity of substitution is not too small.

The right side in Eq. (11) can be also interpreted as the investment elasticity of human capital. Specifically, Eq. (11) can be re-written as

$$\underbrace{-c_3 \frac{u''(c_3)}{u'(c_3)} \frac{\partial H/\partial i_2}{H/i_2}}_{\substack{\text{Human-capital investment} \\ \text{elasticity of marginal utility}}} < \underbrace{\frac{\partial H_1/\partial i_2}{H_1/i_2}}_{\substack{\text{Human-capital investment} \\ \text{elasticity of marginal human capital}}} \tag{13}$$

The left side of Eq. (13) represents human-capital investment elasticity of marginal human capital. Marginal human capital measures how much previous human-capital accumulation contributes to today's human capital stock. The investment in human capital (i_2) increases the future consumption (c_3) and, thus, decreases the marginal utility $(u'(c_3))$. On the other hand, the HCI increases marginal return of previous human-capital. This means that, the increase in the HCI (i_2) makes the previous human capital accumulation (h_2) more effective in producing real income. The condition in (11) shows that if the investment in human capital increases the marginal human capital (H_1) more effectively than the decrease in marginal utility due to increased consumption, the consumer has a positive commitment incentive.

In the case of financial asset accumulation, the previous financial wealth does not affect the return of today's investment because interest rates of savings are exogenously given in the market. Similar to financial assets, part-time labor work, which is not connected to future job career, has no influence on future wage. Assuming that h is part-time labor experience and i is current part-time working hours, the human-capital accumulation function should be $H(h,i) = w \times i$ where w is exogenously given wage, which implies that $H_1 = 0$ (i.e., the past experience in part-time job has no impact on current wage.). The result in Proposition 2 implies that if the past human-capital accumulation does not contribute to the return of income today, the consumer has *negative* commitment incentive to invest in human capital.

4. Conclusion

This paper shows that human-capital investment can be used as a commitment tool if the marginal productivity return of past human-capital accumulation is sufficiently large compared to the diminishing marginal utility of consumption. This property of increasing return of human capital is unique from that of financial assets or labor work. Our results generate an interesting – and perhaps counterintuitive – prediction that present-biased individuals can sometimes have stronger incentive to invest in human capital today, in order to incentivize themselves to invest more in the future. We also show that if the marginal productivity return is not sufficiently large, or it is zero, the consumer can also have negative commitment incentive to invest in human capital. This result departs from the conventional understanding of hyperbolic discounting in which investments in illiquid assets are thought to usually have positive commitment value (e.g. Laibson 1997), and may potentially have interesting real-world applications.

Appendices

A. Multiplicatively-separable utility

In this section, we show that the main results of the paper also hold for multiplicatively-separable utility functions (such as Cobb-Douglas utility functions, which is commonly used in the human-capital literature). Specifically, we define period-utility as

$$u(c_t, l_t) = f(c_t)g(l_t), \tag{14}$$

where $f(c_t)$ and $g(l_t)$ are continuously differentiable, strictly increasing, and strictly concave.

Defining the maximization problem in the same way, the period-2 self will solve the following problem given the human-capital level h_2:

$$\max_{i_2|h_2} f(h_2)g(1-i_2) + \beta\delta \left\{ f(h_3)g(1) \right\}. \tag{15}$$

The first-order condition from the maximization problem of (15) is

$$-f(h_2)g'(1-i_2) + \beta\delta f'(h_3)g(1)H_2(h_2,i_2) = 0. \tag{16}$$

From Eq. (16), we can obtain i_2 as a function of h_2, that is $\widehat{i_2}(h_2) : \mathbb{R}_{++} \to [0,1]$. Plugging the function, $\widehat{i_2}(h_2)$, into the period-1 maximization problem we have

$$\max_{i_1} \begin{bmatrix} f(h_1)g(1-i_1) \\ +\beta\delta \left\{ f(h_2)g(1-\widehat{i_2}(h_2)) + \delta f(h_3)g(1) \right\}. \end{bmatrix} \tag{17}$$

By the condition that $\lim_{i_t \to \infty} H_2(h_t, i_t) = 0$ and $\lim_{i_t \to 0} H_2(h_t, i_t) = \infty$, we know that the consumer does not choose either $i_1 = 0$ or $i_2 = \infty$, so there exists an interior solution i_1.

The first-order condition from the maximization problem in Eq. (17) is

$$\begin{aligned} &-f(h_1)g'(l_1) \\ &+\beta\delta \left\{ f'(h_2)g(l_2)H_2(h_1,i_1) - f(h_2)g'(l_2)\widehat{i'_2}(h_2)H_2(h_1,i_1) \right\} \\ &+\beta\delta^2 f'(h_3)g(1)H_1(h_2,\widehat{i_2}(h_2))H_2(h_1,i_1) \\ &+\beta\delta^2 f'(h_3)g(1)H_2(h_2,\widehat{i_2}(h_2))\widehat{i'_2}(h_2)H_2(h_1,i_1) = 0. \end{aligned} \tag{18}$$

Remembering Eq. (16), we have

$$\beta\delta f'(h_3)g(1)H_2(h_2,i_2) = f(h_2)g'(l_2). \tag{19}$$

From Eqs. (6) and (7), we have

$$\begin{aligned} f(h_1)g'(l_1) \quad = \quad & \underbrace{\begin{matrix} \beta\delta u'(h_2)g(l_2)H_2(h_1,i_1) \\ +\beta\delta^2 u'(h_3)g(1)H_1(h_2,i_2)H_2(h_1,i_1) \end{matrix}}_{\text{Discounted Marginal Utility}} \\ & +\underbrace{(1-\beta)\,\delta\widehat{i'_2}(h_2)H_2(h_1,i_1)f(h_2)g'(l_2)}_{\text{Commitment value}}. \end{aligned} \tag{20}$$

Proposition 1 states that if $\beta < 1$ and $\widehat{i'_2}(h_2) > 0$ (respectively, =0, <0), there is positive (respectively, zero, negative) commitment incentive to invest in human capital. Eq. (20) shows that the result in Proposition 1 also hold for multiplicatively-separable utility functions.

B. Endogenous labor-choice model

Let us assume that the consumer can choose the amount of time they work. In this case, the consumption and leisure in each period become

$$c_t = w_t n_t h_t, \tag{21}$$

and

$$l_t = 1 - n_t - i_t. \tag{22}$$

where n_t is the endogenously chosen labor amount and w_t is exogenously given real wage. In the three-period model, there is no human-capital investment in period 3 so we have $i_3 = 0$.

In this endogenous labor-choice model, the first-order condition in terms of n_t in each period is

$$u'(w_t n_t h_t) w_t h_t - v'(1 - n_t - i_t) = 0. \tag{23}$$

Because u' and v' are strictly decreasing functions, there exists a function of $\overline{n}_t(h_t, i_t)$ that solves the first-order condition of Eq. (23). Because the choice of n_t is not directly affected by other period's choice variables, the choice functions $\overline{n}_t(h_t, i_t)$ are expressed as the the same period decision variables (h_t, i_t). Then, by replacing n_t with $\overline{n}_t(h_t, i_t)$ in all the maximization problems, we can eliminate the variable n_t in the first-order condition in terms of investments. Simply put, because n_t is determined by the balance between the consumption utility and time utility in the same period, the hyperbolic discounting does not directly affect the choice of n_t. Therefore, including an endogenous labor-choice variable does not affect the main result of this paper, which is shown below.

In the endogenous labor-choice model, the period-2 first-order condition in terms of human-capital investment is

$$-v'(l_2) + \beta\delta \left\{ \begin{array}{c} u'(h_3) w_3 H_2(h_2, i_2) n_3 \\ +\underbrace{(u'(w_t n_t h_t) w_t h_t - v'(1 - n_t - i_t))}_{=0 \text{ by } Eq.(23)} \frac{\partial \overline{n}_t(h_t, i_t)}{\partial h_2} H_2(h_2, i_2) \end{array} \right\} = 0, \tag{24}$$

which is equivalent to

$$-v'(l_2) + \beta\delta \left\{ u'(h_3) w_3 H_2(h_2, i_2) n_3 \right\} = 0. \tag{25}$$

In the endogenous labor-choice model, the period-1 first-order condition in terms of human-capital investment is

$$
\begin{aligned}
&-v'(l_1) \\
&+\beta\delta\left\{u'(c_2)w_2H_2(h_1,i_1)n_2 - v'(l_2)\widehat{i}_2'(c_2)H_2(h_1,i_1)\right\} \\
&+\beta\delta^2u'(c_3)w_3H_1(h_2,\widehat{i}_2(h_2))n_3H_2(h_1,i_1) \\
&+\beta\delta^2u'(c_3)w_3H_2(h_2,\widehat{i}_2(h_2))n_3\widehat{i}_2'(h_2)H_2(h_1,i_1) = 0.
\end{aligned} \tag{26}
$$

From Eqs. (25) and (26), we have

$$
v'(l_1) = \underbrace{\begin{array}{c}\beta\delta u'(c_2)w_2H_2(h_1,i_1)n_2 \\ +\beta\delta^2u'(c_3)w_3H_1(h_2,\widehat{i}_2(h_2))n_3H_2(h_1,i_1)\end{array}}_{\text{Discounted Marginal Utility}} + \underbrace{(1-\beta)\,\delta\widehat{i}_2'(h_2)H_2(h_1,i_1)v'(l_2)}_{\text{Commitment value}}. \tag{27}
$$

Eq. (27) indicates that the result of Proposition 1 still holds with the addition of endogenous labor choice. However, the result of Proposition 2 cannot be applied directly because we no longer get concise results for $\widehat{i}_2'(h_2)$ similar to what is shown in Eq. (13). However, this does not harm the intuition conveyed in this paper. Even with an endogenous labor-choice model, we can conclude that with sufficiently large degree of complementarity in the human capital function, we would have positive commitment incentives. For example, with high human capital accumulation function (for example, $\varepsilon \to 0$), we know that f_{12} converges to infinity and a positive commitment value is guaranteed.

Because of the similarity of the first-order conditions for both additively- and multiplicatively-separable utility functions, the same main results hold true with the inclusion of endogenous labor choice in the model with multiplicatively-separable utility functions, such as Cobb-Douglas utility. We can prove it in the same way as shown above.

C. Inclusion of consumption-savings decision

Adding consumption-saving decisions in the model, the consumptions in periods 1, 2, and 3 are

$$c_1 = h_1 - s_1,$$

$$c_2 = h_2 + R_2 s_1 - s_2,$$

and

$$c_3 = h_3 + R_3 s_2,$$

where (s_1, s_2) are savings in periods 1 and 2, and (R_2, R_3) are gross interests rates in periods 2 and 3.

The period-2 maximization problem is

$$\max_{s_2, i_2 | s_1, h_2} u(h_2 + R_2 s_1 - s_2) + v(1 - i_2) + \beta\delta \left\{ u(h_3 + R_3 s_2) + v(1) \right\}. \tag{28}$$

From the maximization problem in Eq. (28), we have the following two first-order conditions:

$$-u'(c_2) + \beta\delta R_3 u'(c_3) = 0 \tag{29}$$

$$-v'(1 - i_2) + \beta\delta u'(c_3) H_2(h_2, i_2) = 0. \tag{30}$$

With the existence of financial markets, we have two response functions, $\widehat{s}_2(s_1, h_2)$ and $\widehat{i}_2(s_1, h_2)$, which solve the two first-order conditions in Eqs. (29) and (30).

Plugging the two functions into the period-1 maximization problem, we have

$$\max_{s_1, i_1} \left[\begin{array}{c} u(h_1 - s_1) + v(1 - i_1) \\ +\beta\delta \left\{ \begin{array}{c} u(h_2 + R_2 s_1 - \widehat{s}_2(s_1, h_2)) + v(1 - \widehat{i}_2(s_1, h_2)) \\ +\delta u(h_3 + R_3 \widehat{s}_2(s_1, h_2)) + \delta v(1) \end{array} \right\} \end{array} \right]. \tag{31}$$

The first-order condition of the period-1 maximization problem in terms of i_1 is

$$\begin{aligned} & -v'(l_1) \\ & +\beta\delta \left\{ u'(c_2) H_2(h_1, i_1) - v'(l_2) \widehat{i_2'}(s_1, h_2) H_2(h_1, i_1) \right\} \\ & +\beta\delta^2 u'(c_3) H_1(h_2, \widehat{i}_2(s_1, h_2)) H_2(h_1, i_1) \\ & +\beta\delta^2 u'(c_3) H_2(h_2, \widehat{i}_2(s_1, h_2)) \widehat{i_2'}(s_1, h_2) H_2(h_1, i_1) \\ & \underbrace{+\beta\delta \left\{ -u'(c_2) + \delta R_3 u'(c_3) \right\} \frac{\partial \widehat{s}_2(s_1, h_2)}{\partial h_2} H_2(h_1, i_1)}_{\text{Additional term from consumption-savings decisions}} \\ = \; & 0. \end{aligned} \tag{32}$$

From Eqs. (29-30) and Eq. (32), we have

$$v'(l_1) = \underbrace{\beta\delta u'(h_2)H_2(h_1,i_1) + \beta\delta^2 u'(h_3)H_1(h_2,i_2)H_2(h_1,i_1)}_{\text{Discounted Marginal Utility}} \\ + \underbrace{(1-\beta)\,\delta\widehat{i}_2'(h_2)H_2(h_1,i_1)v'(l_2)}_{\text{Commitment value}} \\ + \underbrace{\beta\delta^2 R_3\,(1-\beta)\,u'(c_3)\frac{\partial\widehat{s}_2(s_1,h_2)}{\partial h_2}H_2(h_1,i_1)}_{\text{Additional term from consumption-savings decisions}} \tag{33}$$

As shown in Eq. (33), the commitment incentive exists even with the addition of consumption-savings into the model. However, there is one more term in Eq. (33). An increase in period-1 human capital investment (i_1) affects the period-2 savings, which consequently have impact on the marginal utility. Now, we need to show that the additional term is positive, i.e., it can provide an additional commitment value to the demand of human capital investment. To prove it, we need to prove that the sign of $\partial\widehat{s}_2(s_1,h_2)/\partial h_2$ is positive. Implicitly differentiating Eq. (29) with h_2, we have

$$-u''(c_2) + u''(c_2)\frac{\partial\widehat{s}_2(s_1,h_2)}{\partial h_2} + \beta\delta R_3 u''(c_3)\frac{\partial\widehat{s}_2(s_1,h_2)}{\partial h_2} = 0,$$

which implies that

$$\frac{\partial\widehat{s}_2(s_1,h_2)}{\partial h_2} = \frac{u''(c_2)}{u''(c_2) + \beta\delta R_3 u''(c_3)} > 0. \tag{34}$$

Eq. (34) implies that the more current human capital level is, the more savings are. A high human capital brings more income, which induce consumers save more through an income effect.

References

Abdulkadiroğlu, A., J. Angrist, and P. Pathak (2014). The elite illusion: Achievement effects at boston and new york exam schools. *Econometrica 82*(1), 137–196.

Acemoglu, D. (1996). A microfoundation for social increasing returns in human capital accumulation. *Quarterly Journal of Economics 111*(3), 779–804.

Ainslie, G. (1992). *Picoeconomics: The strategic interaction of successive motivational states within the person.* Cambridge University Press.

Amador, M., I. Werning, and G.-M. Angeletos (2006). Commitment vs. flexibility. *Econometrica 74*(2), 365–396.

Bénabou, R. and J. Tirole (2002). Self-confidence and personal motivation. *Quarterly Journal of Economics 117*(3), 871–915.

Bénabou, R. and J. Tirole (2004). Willpower and personal rules. *Journal of Political Economy 112*(4), 848–886.

Bernheim, B. D., J. Meer, and N. K. Novarro (2016). Do consumers exploit commitment opportunities? evidence from natural experiments involving liquor consumption. *American Economic Journal: Economic Policy 8*(4), 41–69.

Bond, P. and G. Sigurdsson (2018). Commitment contracts. *Review of Economic Studies 85*(1), 194–222.

Bryan, G., D. Karlan, and S. Nelson (2010). Commitment devices. *Annu. Rev. Econ. 2*(1), 671–698.

Cadena, B. C. and B. J. Keys (2015). Human capital and the lifetime costs of impatience. *American Economic Journal: Economic Policy 7*(3), 126–53.

Dale, S. and A. B. Krueger (2011). Estimating the return to college selectivity over the career using administrative earnings data. Technical report, National Bureau of Economic Research.

Dekel, E., B. L. Lipman, and A. Rustichini (2009). Temptation-driven preferences. *Review of Economic Studies 76*(3), 937–971.

DellaVigna, S. and U. Malmendier (2006). Paying not to go to the gym. *american economic Review 96*(3), 694–719.

Fang, H. and D. Silverman (2009). Time-inconsistency and welfare program participation: Evidence from the nlsy. *International Economic Review 50*(4), 1043–1077.

Gruber, J. and B. Kőszegi (2001). Is addiction "rational"? Theory and evidence. *Quarterly Journal of Economics 116*(4), 1261–1303.

Gruber, J. and B. Kőszegi (2004). Tax incidence when individuals are time-inconsistent: the case of cigarette excise taxes. *Journal of Public Economics 88*(9-10), 1959–1987.

Gruber, J. H. and S. Mullainathan (2005). Do cigarette taxes make smokers happier. *The BE Journal of Economic Analysis & Policy 5*(1).

Gul, F. and W. Pesendorfer (2001). Temptation and self-control. *Econometrica 69*(6), 1403–1435.

Gul, F. and W. Pesendorfer (2004a). Self-control and the theory of consumption. *Econometrica 72*(1), 119–158.

Gul, F. and W. Pesendorfer (2004b). Self-control, revealed preference and consumption choice. *Review of Economic Dynamics 7*(2), 243–264.

Hinnosaar, M. (2016). Time inconsistency and alcohol sales restrictions. *European Economic Review 87*, 108–131.

Jin, L. and M. Kang (2022). Addiction, present-bias, and self-restraint. *Southern Economic Journal 89*(1), 138–159.

Kang, M. and L. S. Ye (2019). Present bias and corporate tax policies. *Journal of Public Economic Theory 21*(2), 265–290.

Kang, M. and L. S. Ye (2021). Can optimism be a remedy for present bias? *Journal of Money, Credit and Banking 53*(1), 201–231.

Kocherlakota, N. R. (2001). Looking for evidence of time-inconsistent preferences in asset market data. *Federal Reserve Bank of Minneapolis Quarterly Review 25*(3), 13–24.

Koo, J. E. and B. H. Lim (2021). Consumption and life insurance decisions under hyperbolic discounting and taxation. *Economic Modelling 94*, 288–295.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics 112*(2), 443–477.

Levitt, S. D., J. A. List, S. Neckermann, and S. Sadoff (2016). The behavioralist goes to school: Leveraging behavioral economics to improve educational performance. *American Economic Journal: Economic Policy 8*(4), 183–219.

Li, H., C. Mu, J. Yang, et al. (2016). Optimal contract theory with time-inconsistent preferences. *Economic Modelling 52*, 519–530.

Liu, B., Y. Niu, and Y. Zhang (2019). Corporate liquidity and risk management with time-inconsistent preferences. *Economic Modelling 81*, 295–307.

Loewenstein, G. and D. Prelec (1992). Anomalies in intertemporal choice: Evidence and an interpretation. *Quarterly Journal of Economics 107*(2), 573–597.

Newhouse, J. P. (2006). Reconsidering the moral hazard-risk avoidance tradeoff. *Journal of Health Economics 25*(5), 1005–1014.

O'Donoghue, T. and M. Rabin (1999). Doing it now or later. *American Economic Review 89*(1), 103–124.

O'Donoghue, T. and M. Rabin (2001). Choice and procrastination. *Quarterly Journal of Economics 116*(1), 121–160.

O'Donoghue, T. and M. Rabin (2006). Optimal sin taxes. *Journal of Public Economics 90*(10-11), 1825–1849.

Oreopoulos, P. (2007). Do dropouts drop out too soon? wealth, health and happiness from compulsory schooling. *Journal of Public Economics 91*(11-12), 2213–2229.

Phelps, E. S. and R. A. Pollak (1968). On second-best national saving and game-equilibrium growth. *Review of Economic Studies 35*(2), 185–199.

Pollak, R. A. (1968). Consistent planning. *Review of Economic Studies 35*(2), 201–208.

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy 94*(5), 1002–1037.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy 98*(5, Part 2), S71–S102.

Ruhm, C. J. (2012). Understanding overeating and obesity. *Journal of Health economics 31*(6), 781–796.

Saint-Paul, G. (1996). Unemployment and increasing private returns to human capital. *Journal of Public Economics 61*(1), 1–20.

Schilbach, F. (2019). Alcohol and self-control: A field experiment in india. *American economic review 109*(4), 1290–1322.

Schleich, J., X. Gassmann, T. Meissner, and C. Faure (2019). A large-scale test of the effects of time discounting, risk aversion, loss aversion, and present bias on household adoption of energy-efficient technologies. *Energy Economics 80*, 377–393.

Strotz, R. H. (1956). Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies 23*(3), 165–180.

Thaler, R. (1981). Some empirical evidence on time inconsistency. *Review of Economic Studies 23*, 165–180.

Werthschulte, M. and A. Löschel (2021). On the role of present bias and biased price beliefs in household energy consumption. *Journal of Environmental Economics and Management 109*, 102500.