Dividend and corporate income taxation with present-biased consumers

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Abstract

Debates on the double taxation of dividends and corporate income taxes have been long-standing. If double taxation were to be avoided, which type of taxation policy would be more ideal? Conventional corporate theory based on microeconomic approaches does not yield a definitive answer to this question, as either policy would distort firm investment and decrease firm value. Distinct from previous models, this paper addresses the double taxation issue in a macroeconomic context under a Laibson's hyperbolic discounting model. In particular, this paper shows that dividend taxation can improve consumer welfare even though it decreases firm value in the hyperbolic economy. On the other hand, corporate taxes have a negative impact on both consumers and firms.

Keywords: Corporate tax policy; Dividend tax; Profit tax; Present-biased preferences; Hyperbolic discounting subsidy

JEL codes: H25, G38, E22.

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1. Introduction

When dividend taxes are imposed in conjunction with corporate income tax, a double taxation of corporate profits can arise and this has been a source of long-standing debate. Under double taxation, two types of taxes are levied on the same corporate income source for shareholders. Policymakers often face a choice on the relative weight placed on these two taxation policies. In the United States, tax reforms in past twenty years have adjusted both the dividend and corporate income tax rates. For example, the dividend tax was reduced to 15 percent in 2003, while nearly 15 years later, the corporate profits tax was reduced to an effective rate of 21 percent.[1] In other countries, policies have sought to integrate these two taxes, such as corporate tax deductions of dividends. These varied policies show that there is no consensus on the relative costs and benefits of these two taxation policies.

If double taxation were to be avoided, which tax policy would be ideal from a policy standpoint? Another important question is can corporate taxes benefit the macroeconomy as a whole even though they decrease firm value? This paper addresses these questions based on Laibson's (1997) hyperbolic discounting macroeconomic model. To the best of our knowledge, this is the first paper to address the corporate double taxation issue in the general equilibrium context. Most of the previous models on this issue are restricted to the pure corporate setting. Depending on the choice of market friction in the microeconomic context, conclusions about the choice of corporate taxes can vary. For example, Chetty and Saez (2010)'s model is a two-period corporate agency setting where the manager can invest in unproductive projects (i.e., "pet" project), which would increase the manager's utility but not the shareholder's utility. They show that corporate income taxes can help curb investment in "pet" projects by increasing shareholders' wealth. On the other hand, Kang and Ye (2019) show that dividend taxes should be chosen to improve firm value if the manager has more short-term oriented preferences than the owner. Those papers are modeled based on agency problems but lead to different conclusions.

[1] The corporate finance literature has shown that government's corporate tax policies can significantly affect corporate decisions. Specifically, the 2003 dividend tax cut significantly affected corporate payouts decision (see Brown, Liang and Weisbenner 2007, Brav et al. 2008, Hanlon and Hoopes 2014). Moreover, the managerial decisions in response to dividend tax cuts also significantly affected the US stock market and firms' expected returns (see Dhaliwal, Li and Trezevant 2003 and Amromin, Harrison and Sharpe 2008).

We assume that the firm is rational, so any government intervention would decrease firm value.[2] However, we consider this corporate taxation problem from a macroeconomic perspective. We examine whether reduced firm value from corporate taxes necessarily implies a negative effect for the macroeconomy. In the macroeconomic context, the owner of firms are consumers such that if they are better off from the tax policy, we can say that some form of corporate tax policy could be welfare-improving even though firm value decreases.

To understand the overall welfare impact of corporate taxes, we use Laibson's hyperbolic discounting model. Vast empirical and experimental evidence indicates that human behavior follows hyperbolic rather than exponential discounting.[3] Based on this evidence, Strotz (1956) and Phelps and Pollak (1968), and Laibson (1997) constructed intertemporal consumption-savings decision models based on present-biased preferences. Their previous research showed that under (quasi-) hyperbolic discounting preferences, there are undersaving problems due to consumers' self-control problems. In particular, Phelps and Pollak (1968) showed that marginal increases in equilibrium savings can improve all intertemporal utilities. Laibson (1997) also indicated that in a complete financial market, it is impossible for consumers to escape undersaving traps, even if they are aware of their self-control problems.

The natural next question is how outside authorities such as governments can help amend this undersaving problem. There is considerable research that suggests a role for tax policies. One of the main policy tools is the savings subsidy policy (i.e., a capital subsidy or interest subsidy), which decreases the cost of savings, thus inducing consumers to increase equilibrium savings (see Laibson 1996, Krusell, Kuruşçu and Smith, 2010, and Pavoni and Yazici, 2017).[4] Even though there is a large literature investigating tax policy on consumers, these papers have not investigated the impact

[2]There is limited evidence that a corporation makes decisions based on present biased preferences, but there is much evidence that consumers are present biased. As Grenadier and Wang (2009) mentioned, it is unlikely that large corporations are present biased because professional managers can mitigate the time-inconsistency from firms' decisions. DellaVigna and Malmendier (2004) also assumed that consumers are time-inconsistent, but that firms are rational.

[3]See Thaler (1981), Loewenstein and Prelec (1992) and Frederick, Loewenstein and O'Donoghue (2002) for discussions on present-biased preferences.

[4]In the representative agent macroeconomics model with complete markets, the amount of aggregate capital is considered to be the same as that of aggregate savings. Therefore, capital subsidy and savings subsidy are identical policies in the model. Under capital subsidies (i.e., savings subsidies) the consumer is subsidized when purchasing capital, while under interest subsidies she is subsidized when liquidating capital. Therefore, the three types of subsidies are isomorphic or identical.

of corporate taxes under a hyperbolic economy.

To model corporate tax policy in a macroeconomy, this paper incorporates a corporation's dividend-investment decision model into a representative macroeconomics model. The firm maximizes the present value of dividend payouts and decides on the investment amount and dividend payout. Even if the firm behaves rationally, the firm will not hold enough capital stock if present-biased consumers save less through the stock and bond markets. These consumers' undersavings through the capital markets will result in firms being short on cash, which causes an underinvestment problem.

The main conclusion of this paper is that both corporate income taxes and dividend taxes decrease firm value but the dividend tax can improve consumer welfare. The following is the mechanism for how a dividend tax can boost consumers' welfare. Dividend taxes increase the firm's cost of dividend payout and thereby decreases the relative cost of investment. Therefore, the dividend tax increases firms' demand for investment and thus the equilibrium interest rate. The increase in the interest rate has an effect equivalent to an increase in the compensated interest rate, because firms' higher investment level induced by government intervention decreases consumers' dividend income. Since the lower dividend income offsets the increased savings income due to the higher interest rate, the substitution effect that increases savings plays a role in consumers' decisions. Therefore, the subsidy policy eventually induces consumers to save more through the capital market. On the other hand, corporate income taxes decrease firms' demand for investment and thus the equilibrium interest rate. The lower interest rates induce consumers to save less, which lead to the undersaving problem and lower welfare.

This paper shows that there is always a Pareto-improving dividend tax policy under a revenue-neutral regime. We assume that the collected dividend tax is distributed as lump-sum subsidies collected to consumers or corporations, so there is no change in government revenue as a result of the tax-subsidy policy. The main difficulty in designing a welfare-improving tax-subsidy policy under hyperbolic discounting is that a one-period tax policy decreases that period's intertemporal utility. This is because the consumer rationally maximizes the intertemporal utility in each period, which is viewed as biased from the perspective of other periods but not from that of the current period. However, we show that the policy in one period can improve the other periods' intertemporal utilities, so the combination of all periods' policies can Pareto-improve the equilibrium allocations.

The rest of the paper is organized as follows. Section 2 introduces a macroeconomic model in which the representative firm makes dividend-investment decisions. Section 3 shows that the dividend tax policy can Pareto-improve the equilibrium allocations in the three-period model. Section 4 shows the negative effect from corporate income taxes under the hyperbolic economy. In Appendix A, this paper presents a leading example, to help explain the main results in this paper. All the proofs of propositions and lemmas are in the Appendices.

2. The model

This section introduces a three-period model in which a representative consumer maximizes hyperbolically-discounted intertemporal utilities and a representative firm maximizes the present value of dividend payouts. To incorporate time-inconsistency into the macroeconomic model, we need at least three periods.

2.1. A representative firm

We define a representative firm's production function in period t as $A_tF(K_t, N_t)$, where A_t, K_t, and N_t represent the period-t total factor productivity, aggregate capital, and aggregate labor, respectively. Function $F(K_t, N_t)$ satisfies the Inada conditions and exhibits constant returns to scale. We define per-worker production function as $f_t(k_t) = A_tF(K_t, N_t)/N_t$, where $k_t\,(= K_t/N_t)$ is the per-capita capital in period t. Assume that there exists a continuum of individual agents indexed by the unit interval. Individual decision and state variables are represented by an i index. Labor is assumed to be supplied inelastically, so $N_t = \int_0^1 N_t(i)di = 1$.

In each period, the firm makes decisions on the dividend payout and investment. If the firm invests I_t amount of capital goods in period t, the capital in period $t+1$ would be $K_{t+1} = K_t(1-d) + I_t$ where $d \in (0, 1)$ represents the capital depreciation rate. In period 0, the firm is endowed with K_0 units of capital and indebted with b_{-1} units of bonds.

Denote v_t as the dividend payout in period t; the the dividend payout in periods 0, 1, 2 would be

$$v_0 = A_0F(K_0, N_0) - I_0 - w_0N_0 - R_0b_{-1} + b_0, \tag{1}$$

$$v_1 = A_1 F(K_1, N_1) - I_1 - w_1 N_1 - R_1 b_0 + b_1, \tag{2}$$

$$v_2 = A_2 F(K_2, N_2) - w_2 N_2 - R_2 b_1 + (1-d)(1-\chi)K_2 \tag{3}$$

where

v_t : dividend payout,	w_t : real wage,
b_t : corporate bond issue,	R_t : real gross interest rate,
I_t : investment,	χ : capital liquidation cost.

In periods 0 and 1, the firm makes decision on dividends and investments as shown in *Eqs.* (1) and (2). To ensure that the firm makes a positive amount of investment in each period (i.e., so the firm has no incentive to liquidate the capital), we assume that there is sufficient technological improvement such that $A_0 < A_1 < A_2$. In period 2, which is the last period, the firm liquidates all the capital with a proportional liquidation cost, χ, as shown in *Eq.* (3). The firm maximizes the present value of dividend payouts (i.e., firm value). The present values of dividend payouts are $v_0 + v_1/R_1 + v_2/(R_1R_2)$ in period 0, $v_1 + v_2/R_2$ in period 1, and v_2 in period 2, respectively.

The period-2 firm maximizes the present value of dividend payouts:

$$\max_{N_2} v_2. \tag{4}$$

Given the capital level (K_2), the firm makes decisions based on the labor choice, so the first-order condition from the maximization problem of *Eq.* (4) is

$$w_2 = A_2 F_2(K_2, N_2).$$

The period-1 firm maximizes its present value of the dividend payouts:

$$\max_{N_1, I_1, b_1} v_1 + v_2/R_2. \tag{5}$$

The first-order conditions in terms of labor and investment, respectively, from the period-1 maximization problem of *Eq.* (5) are

$$w_1 = A_1 F_2(K_1, N_1) \tag{6}$$

and

$$R_2 = A_2 F_1(K_2, N_2) + (1-d)(1-\chi). \tag{7}$$

The supply for corporate bonds (b_t) is perfectly inelastic in the maximization problem of *Eq.* (5), so the equilibrium quantity of bonds are determined by the consumer demand for bonds.

The firm's maximization problem in period 0 is

$$\max_{N_0, I_0, b_0} v_0 + v_1/R_1 + v_2/(R_1 R_2). \tag{8}$$

The first-order conditions from the period-0 maximization problem of *Eq.* (8) are

$$w_0 = A_0 F_2(K_0, N_0), \tag{9}$$

and

$$R_1 = A_1 F_1(K_1, N_1) + (1-d)\frac{A_2 F_1(K_2, N_2) + (1-d)(1-\chi)}{R_2}. \tag{10}$$

From *Eq.* (7) and (10), we can derive the firm's demand for period-0 investment:

$$R_1 = A_1 F_1(K_1, N_1) + (1-d). \tag{11}$$

In this paper, we assume that the firm issues bonds but not shares of stock. However, including the stock market does not change the main result of this paper because in the complete market, the effective values of stock and bonds should be the same.

2.2. A representative consumer

A representative consumer lives in three periods, $t = 0, 1, 2$. The consumer's period utility $u(c)$ is strictly increasing, strictly concave, twice continuously differentiable and $\lim_{c \to 0} u'(c) = \infty$, where c is a perishable consumption good. We assume that the representative consumer has ownership of the firm and is endowed with b_{-1} units of corporate bonds in period 0. The consumer is endowed with one unit of labor good, which has an inelastic supply. Thus, the consumer's resource constraints are

$$c_0 + b_0 = w_0 + v_0 + R_0 b_{-1}, \tag{12}$$

$$c_1 + b_1 = w_1 + v_1 + R_1 b_0, \tag{13}$$

and

$$c_2 = w_2 + v_2 + R_2 b_1, \tag{14}$$

in periods 0, 1 and 2, respectively, where b_t, w_t, and v_t are the amount of bond holding, real wage, and the dividend income (i.e., dividend payout), respectively, in period t.[5]

The consumer's intertemporal utilities in the three periods, $U^{(0)}, U^{(1)}$, and $U^{(2)}$ are

$$U^{(0)}(c_0, c_1, c_2) = u(c_0) + \beta \left(\delta u(c_1) + \delta^2 u(c_2)\right),$$

$$U^{(1)}(c_1, c_2) = u(c_1) + \beta\delta u(c_2),$$

and

$$U^{(2)}(c_2) = u(c_2),$$

where $\delta \in (0,1)$ is a long-run discounting factor and $\beta \in (0,1)$ is a hyperbolic discounting factor. If $\beta = 1$, the consumer's preference follows exponential discounting and thus would be a time-consistent decision maker. If $\beta < 1$, the consumer follows quasi-hyperbolic discounting and thus would be time-inconsistent.

The consumer perfectly forecasts future market prices, labor income and dividend income. She also knows her future preferences (sophisticated consumer). Therefore, we should solve the maximization problems through backward induction. The period-2 self consumes all her financial and labor incomes so the period-2 intertemporal utility is

$$U^{(2)}(c_2) = u(w_2 + v_2 + R_2 b_1).$$

Given $(R_t, w_t, v_t)_{t=1}^2$, the period-1 self solves the following maximization problem, conditional on b_0 :

$$\max_{b_1 | b_0} U^{(1)} \left(w_1 + v_1 + R_1 b_0 - b_1, w_2 + v_2 + R_2 b_1\right). \tag{15}$$

[5] In *Eq.* (12), the value of R_0 does not affect the consumer's financial income (i.e., $v_1 + R_0 b_{-1}$) because higher R_0 decreases the firm's liability along with the consumer's dividend income v_1.

From the maximization problem of *Eq.* (15), we implicitly derive b_1 as a function of b_0, conditional on $(R_t, w_t, v_t)_{t=1}^2$, denoted as $\bar{b}_1(b_0)$. Given the savings response function $\bar{b}_1(b_0)$ and $(R_t, w_t, v_t)_{t=0}^2$, the consumer chooses b_0 to maximize $U^{(0)}$:

$$\max_{b_0} U^{(0)} \left(\begin{array}{c} w_0 + v_0 + R_0 b_{-1} - b_0, w_1 + v_1 + R_1 b_0 - \bar{b}_1(b_0), \\ w_2 + v_2 + R_2 \bar{b}_1(b_0) \end{array} \right). \tag{16}$$

The consumer's optimal choice of savings can be characterized as a subgame perfect Nash equilibrium $\left(b_0^*, \bar{b}_1(b_0)\right)$ such that $\bar{b}_1(b_0)$ solves the period-1 maximization problem of *Eq.* (15), conditional on b_0; and b_0^* solves the period-0 maximization problem of *Eq.* (16).

2.3. The equilibrium

The equilibrium is characterized by the consumer's maximization problems in *Eqs.* (15) and (16), given $\{(R_t, w_t, v_t)_{t=0}^2, b_{-1}\}$; the firm's maximization problems in *Eqs.* (4), (5), and (8) given $\{(R_t, w_t)_{t=0}^2, b_{-1}\}$; and the labor, and commodity market clearing conditions. In equilibrium, the gross interest rate, real wages, and dividends are given by

$$R_1 = f_1'(k_1) + (1 - d), \tag{17}$$

$$R_2 = f_2'(k_2) + (1 - d)(1 - \chi), \tag{18}$$

$$w_t = f_t(k_t) - k_t f_t'(k_t) \text{ for all } \ t = 0, 1, 2, \tag{19}$$

and

$$v_t = k_t f_t'(k_t) - I_t - R_t b_{t-1} + b_t \text{ for all } \ t = 0, 1, 2. \tag{20}$$

where $f_t(k_t) = A_t F(K_t, N_t)$, $I_2 = 0$, and $b_2 = 0$.

From the firm and consumer's budget constraint in *Eqs.* (1-3) and (12-14), the commodity market clearing condition is reduced to

$$c_t = f_t(k_t) - I_t. \tag{21}$$

The bond market clearing condition is satisfied because we use the same symbol (b_t) for both the consumer's savings and firm's bonds. We have the following labor market condition:

$$\int_0^1 N_t(i)di = 1 = N_t, \text{ for all } \ t = 0, 1, 2.$$

2.4. Undersavings problems

It is a well-known result that present-biased consumers undersave, which means that increasing savings in equilibrium can improve their welfare (see Phelps and Pollak 1968; Goldman 1978; Laibson 1996). In a general-equilibrium model, this undersavings problem results in an underinvestment problem, which means that the economy does not have sufficient capital stock to optimize welfare. This section addresses this undersaving/underinvestment problem in the three-period model. Using the commodity market clearing conditions of *Eq.* (21) and $k_{t+1} = k_t(1-d) + I_t$, the equilibrium is characterized by two capital levels, i.e., (k_1, k_2). Thus, in equilibrium, the intertemporal utilities can be characterized as

$$\begin{aligned}\overline{U}^{(0)}(k_1, k_2) \ &= \ U^{(0)}(f_0(k_0) + (1-d)k_0 - k_1, \\ &\quad f_1(k_1) + (1-d)k_1 - k_2, f_2(k_2) + (1-d)(1-\chi)k_2),\end{aligned}$$

$$\overline{U}^{(1)}(k_1, k_2) = U^{(1)}(f_1(k_1) + (1-d)k_1 - k_2, f_2(k_2) + (1-d)(1-\chi)k_2)$$

and

$$\overline{U}^{(2)}(k_1, k_2) = U^{(2)}(f_2(k_2) + (1-d)(1-\chi)k_2),$$

where $\overline{U}^{(0)}(k_1, k_2)$, $\overline{U}^{(1)}(k_1, k_2)$ and $\overline{U}^{(2)}(k_1, k_2)$ are the intertemporal utilities in terms of (k_1, k_2).

Figure 1 describes the equilibrium capital levels (k_1^*, k_2^*) without any government intervention in the example with $f_i(k) = 10\sqrt{k}$ for i=0,1,2, $u(c) = \ln c$, $\delta = 1, \beta = 1/2, d = 100\%$, and $k_0 = 10$. As shown in Figure 1, the indifference curve of period-0 utility, $\overline{U}^{(0)}(k_1, k_2)$, passing through the equilibrium capital level has a circular shape, while the indifference curve of period-1 utility, $\overline{U}^{(1)}(k_1, k_2)$, has a slanted parabolic shape. The Pareto-superior region is the overlapping area between the two indifference curves, such that intertemporal utility values (i.e., $U^{(0)}, U^{(1)}$, and $U^{(2)}$) associated with any capital level inside this region are higher than those associated with the equilibrium capital level. The Pareto-superior region is located North-East of the equilibrium point, which implies that the economy has an underinvestment

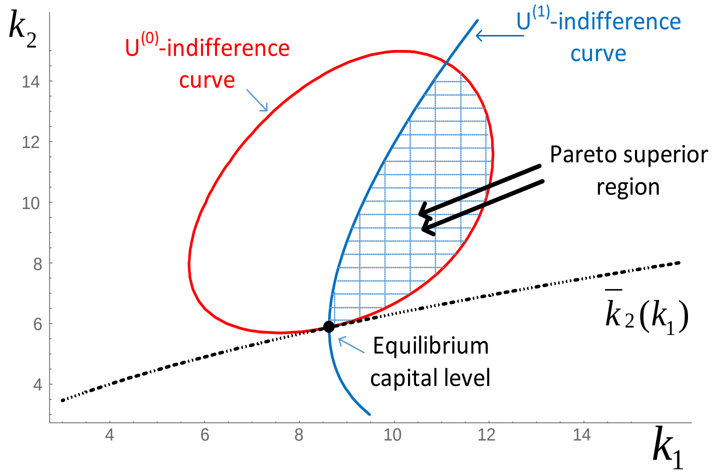

Figure 1: Equilibrium capital level and underinvestment problem without government intervention in the example with $f_i(k) = 10\sqrt{k}$ for i=0,1,2, $u(c) = \ln c$, $\delta = 1, \beta = 1/2, d = 100\%$, and $k_0 = 10$.

problem.[6] For detailed proof of the existence of the underinvestment problem with general utility/production functions, see Appendix E.

In Figure 1, the capital response function $\bar{k}_2(k_1)$ represents the equilibrium capital in period 2 (k_2) given period-1 capital level k_1. The capital response function $\bar{k}_2(k_1)$ is different from the savings response function $\bar{b}_1(b_0)$ but the first derivatives of the two functions are the same, i.e., $\bar{k}_2'(k_1) = \bar{b}_1'(b_0)$.

3. Dividend tax policy

In the previous section, we introduced a general equilibrium model where the firm makes dividend-investment decisions and showed that the economy would experience underinvestment problem without any government intervention. This section introduces the Pareto-improving dividend tax policies.

[6] As β approaches one (i.e., the consumer becomes more time consistent), the Pareto-superior area shrinks and converges to the equilibrium point.

3.1. The model with dividend tax

With a dividend tax policy, the modified firm's budget constraint in periods 1 and 2 are

$$v_0\,(1+\tau_0) = A_0F(K_0,N_0) - I_0 - w_0N_0 - R_0b_{-1} + b_0 + S_0, \tag{22}$$

$$v_1\,(1+\tau_1) = A_1F(K_1,N_1) - I_1 - w_1N_1 - R_1b_0 + b_1 + S_1, \tag{23}$$

where $\tau_t > 0$ is the proportional dividend tax rate and $S_t > 0$ is a lump-sum tax in period t.[7] Under a revenue-neutral policy, the budget constraints satisfy $S_t = \tau_t v_t^*$ where v_t^* is the equilibrium dividend level in period t. In *Eqs.* (22) and (23), the lump-sum subsidy is applied to the corporation rather than the consumer. However, even if the lump-sum subsidy is imposed on the consumer, there would be no change in the equilibrium allocations. In this case, the consumer's increased income will be compensated with lower dividend income; thus, there would no change in the consumer's after-tax income.

Under dividend taxation, the first-order conditions from the firm's period-1 maximization problem of *Eq.* (5) are

$$w_1 = A_1F_2(K_1,N_1) \tag{24}$$

and

$$\frac{1}{1+\tau_1} = \frac{A_2F_1(K_2,N_2) + (1-d)(1-\chi)}{R_2}. \tag{25}$$

The first-order conditions from the period-0 maximization problem of *Eq.* (8) is

$$w_0 = A_0F_2(K_0,N_0), \tag{26}$$

and

$$\frac{1}{1+\tau_0} = \frac{A_1F_1(K_1,N_1)}{R_1}\frac{1}{1+\tau_1} + (1-d)\frac{A_2F_1(K_2,N_2) + (1-d)(1-\chi)}{R_1R_2}. \tag{27}$$

[7] The dividend tax in our model is not applied in the last period ($t = 2$). Because in the last period ($t = 2$), the firm does not need to have investment (zero investment), the dividend policy does not affect the investment decision. That is, in the last period, the dividend payout is the same as corporate income. Therefore, under revenue neutral policy, the dividend tax does not affect the equilibrium outcome in period $t = 2$. To simplify the model, we did not consider the dividend tax in period 2. However, adding the tax, the main result is invariant.

From *Eq.* (25) and (27), we can derive the firm's demand for period-0 investment:

$$\frac{1}{1+\tau_0} = \left(\frac{A_1 F_1(K_1, N_1)}{R_1} + \frac{(1-d)}{R_1}\right)\frac{1}{1+\tau_1}. \tag{28}$$

In this economy, there exists an equilibrium, which is shown in the following lemma:

Lemma 1 *There exists an open set of $T \subset \mathbb{R}^2$ such that $T \ni (0,0)$ and for any $(\tau_0, \tau_1) \in T$, an equilibrium of this economy exists.*

Lemma 1 suggests that there exists an equilibrium with zero tax (or $(\tau_0, \tau_1) = (0,0)$) and with moderate levels of tax policies around zero subsidy. However, if the values of the taxes, (τ_0, τ_1), are too large, the dividend can diverge to a large negative value due to low investment costs, which results in negative income and thus equilibrium would become nonexistent. Therefore, this paper analyzes the impact of infinitesimal increases in subsidies from zero to small positive values. The equilibrium described in Lemma 1 satisfies the consumer's intrapersonal subgame described in subsection 2.2 and the firm's profit maximization problems in subsection 2.1.

3.2. Pareto-improving dividend tax policies

This subsection shows that dividend tax policy can resolve the underinvestment problem. Specifically, we show that there is always a tax plan (τ_0, τ_1) that moves the equilibrium capital level into the Pareto-superior region in Figure 1.

The result in Lemma 1 implies that where $(\tau_0, \tau_1) = (0,0)$, there is an equilibrium that satisfies the first- and second-order conditions. This also implies that for infinitesimal variations of tax rates from zero to a small positive value, there is still equilibrium. In the following lemma, we investigate how an infinitesimal increase in τ_0 affects the equilibrium capital level and welfare:

Lemma 2 *At an equilibrium with $(\tau_0, \tau_1) = (0,0)$, a marginal increase in τ_0 increases the equilibrium capital level in both periods and the following equality is satisfied:*

$$\frac{dk_2^*}{d\tau_0} / \frac{dk_1^*}{d\tau_0} = \overline{k}_2'(k_1) = \overline{b}_1'(b_0).$$

This results in a decrease in the period-0 intertemporal utility but an increase in the future intertemporal utilities, so we have

$$\frac{dU^{(0)}}{d\tau_0} < 0, \frac{dU^{(1)}}{d\tau_0} > 0 \text{ and } \frac{dU^{(2)}}{d\tau_0} > 0. \tag{29}$$

This also results in a decrease in the period-0 firm value.

First, it is straightforward by envelope theorem to show that the increase in τ_0 decreases period-0 firm value in Lemma 2. Lemma 2 also indicates that both k_1 and k_2 increase with a period-0 tax. Higher τ_0 increases the firm's demand for period-0 investment, and thus increases the period-1 real interest rate. The increased real interest rate makes consumers save more (i.e., increases b_0) so that the capital level k_1 increases. However, Lemma 2 also indicates that the equilibrium allocations are not Pareto improving only with a period-0 tax. The current consumer (in period 0) maximizes her intertemporal utility, which is rational based on the current consumer's perspective but is present biased based on the future consumer's perspective. Therefore, the current tax policy, which distorts the current interest rate, would make the current consumer worse off but would make consumers in other periods better off.

To move the equilibrium capital level into the Pareto-superior region, the increasing ratio of k_2 relative to k_1 should be greater than $\overline{k}_2'(k_1)$, which can be achieved with a period-1 tax. In the following lemma, we show how the period-1 tax affects the equilibrium investment plan:

Lemma 3 *At the equilibrium with* $(\tau_0, \tau_1) = (0, 0)$*, a marginal increase in* τ_1 *satisfies the following inequality:*

$$\overline{k}_2'(k_1)\frac{dk_1}{d\tau_1} < \frac{dk_2}{d\tau_1}, \tag{30}$$

which implies that there is an increase in the period-0 intertemporal utility, i.e.,

$$\frac{dU^{(0)}}{d\tau_1} > 0. \tag{31}$$

This also results in a decrease in the period-1 firm value.

In Lemma 3, the inequality in *Eq.* (30) means that τ_1 increases the capital response function $\overline{k}_2(k_1)$, which means that any given k_1, k_2 with $\tau_1 > 0$ is greater than that with $\tau_1 = 0$. Therefore, Lemma 3 also implies that an increase in period-1

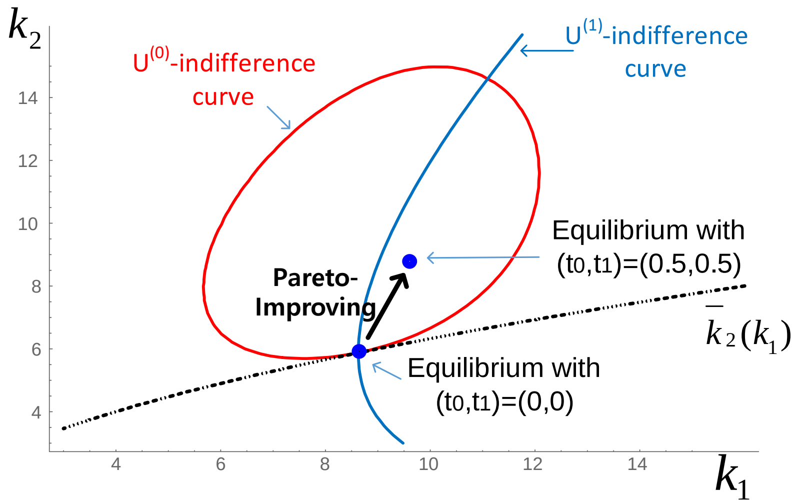

Figure 2: A Pareto-improving corporate tax policy with $(\tau_1, \tau_2) = (0.5, 0.5)$ in the example with $f_i(k) = 10\sqrt{k}$ for i=0,1,2, $u(c) = \ln c$, $\delta = 1, \beta = 1/2, d = 100\%$, and $k_0 = 10$.

tax increases the equilibrium real interest rate and thus also increases the period-1 savings level (b_1) for any given b_0. This increase in future savings (b_1) increases the future capital level (k_2), which has a positive impact on period-0 intertemporal utility.

From Lemmas 2 and 3, we can show the existence of Pareto-improving investment subsidy policies.

Proposition 1 *There exist dividend tax policies* $(\tau_0, \tau_1) \gg 0$ *that improve all intertemporal utilities.*

Proposition 1 indicates that implementing the tax policy in both periods can Pareto-improve the equilibrium allocations. $U^{(1)}$ and $U^{(2)}$ improve with a period-0 tax but $U^{(0)}$ does not. However, together with period-0 and period-1 taxes, all intertemporal utilities can improve. As shown in Figure 2, positive dividend taxes can move the equilibrium allocation to move inside the Pareto-superior region in the given example. For detailed numerical results of the example, see Appendix A.

The two most prevalent welfare criteria for hyperbolic discounting models are the Pareto criterion that takes into account all periods' intertemporal utilities and the long-run perspective criterion that considers the intertemporal utility in fictitious period -1. Proposition 1 uses the Pareto criterion, which takes into account intertemporal utilities across all periods. On the other hand, O'Donoghue and Rabin (1999,

2015) proposed long-run perspective preferences that are determined in period -1. Specifically, in our model, the long-run perspective preferences is

$$U^{(-1)} = \beta \left\{ \delta u(c_0) + \delta^2 u(c_1) + \delta^3 u(c_2) \right\}, \tag{32}$$

which is affinely equivalent to time-consistent preferences with a discounting factor of δ. As shown in Kang (2015, 2019), for general T-period time-separable utility with quasi-hyperbolic discounting, any policy improving intertemporal utilities across all periods also improves the long-term utility. Therefore, the policy proposed in Proposition 1 improves the long-run perspective preference utility, $U^{(-1)}$, as well.

4. Corporate income tax

In this section, we show that the corporate income tax deteriorates the under-savings problem and thus decreases both firm value and consumer welfare. With a corporate income tax policy, the modified firm's budget constraint in periods 1 and 2 are

$$v_1 = A_1 F(K_1, N_1)(1-\theta_1) - I_1 - w_1 N_1 - R_1 b_0 + b_1 + M_1, \tag{33}$$

$$v_2 = A_2 F(K_2, N_2)(1-\theta_2) + (1-d)(1-\chi)K_2 - w_2 N_2 - R_0 b_1 + M_0, \tag{34}$$

where $\theta_t > 0$ is a proportional corporate income tax rate and $M_t > 0$ is a lump-sum subsidy in period t. Under a revenue-neutral policy, the budget constraints satisfy $M_t = \theta_t f_t(k_t^*)$, where k_t^* is the equilibrium capital level in period t.

From the firm's period-0 first-order conditions in terms of investment, we have

$$R_2 = A_2 F_1(K_2, N_2)(1-\theta_2) + (1-d)(1-\chi). \tag{35}$$

From the first-order conditions of the period-0 maximization problem, we have

$$\begin{aligned} R_1 &= A_1 F_1(K_1, N_1)(1-\theta_1) \\ &\quad + (1-d)\frac{A_2 F_1(K_2, N_2)(1-\theta_2) + (1-d)(1-\chi)}{R_2}. \end{aligned} \tag{36}$$

From *Eq.* (35) and (36), we can derive the firm's inverse demand function for period-0 investment:

$$R_1 = A_1 F_1(K_1, N_1)(1-\theta_1) + (1-d). \tag{37}$$

From Eqs. (35) and (37), we know that the corporate income taxes, in general, decrease the interest rate, which push down the consumer's demand for savings. Thus, the underinvestment problem under hyperbolic discounting is larger with corporate income taxes and consumer welfare decreases.

Following the same but reverse logic as the dividend-taxation case in Section 4, an increase in θ raises the cost of investment relative to the cost of dividends.

Proposition 2 *There exist corporate income tax policies* $(\theta_0, \theta_1) \gg 0$ *that decrease all intertemporal utilities.*

Even though dividend taxation and corporate income taxation are structurally different under this setting, to prove the negative impact on welfare from income tax in Proposition 2, the same but reverse logic as that of proof of Proposition 1 is applied. The increase in corporate income tax decreases the cost of dividend payout and raises the cost of investment, which is mathematically equivalent to the case of a decrease in dividend taxes. As corporate income tax increases, the investment level moves to the Pareto-inferior region, where the value of the firm for all periods is lower than the Nash equilibrium value without tax policy. This also implies that corporate income subsidies necessarily Pareto improve firm value.

5. Conclusion

This paper addresses the double taxation issue in a macroeconomic context under a Laibson's hyperbolic discounting model. We show that dividend taxation can improve consumer welfare even though it decreases firm value in the hyperbolic economy, while corporate income taxes have a negative impact on both consumers and firms.

In this framework, the corporation has ownership of the capital, but the consumer has ownership of the corporation in the form of bonds and stocks. The consumers indirectly affect the corporation's budget through the stock and bond markets. In this paper, the underinvestment problem is caused by consumers' low demand for corporate bonds, which would make it difficult for the firm to hold enough cash to fund investments. Therefore, even though the firm is a rational decision maker, the economy with hyperbolic consumers cannot avoid the underinvestment problem.

This paper's approach is better suited to analyzing the linkage between corporate-level decisions and the macroeconomy. Conventional macroeconomic models often assume that the firm rents capital from the consumer every period for simplicity. Under this setting, it would not be possible to model corporate investment decisions or government policy on corporations. The analysis in this paper suggests further research modelling corporate decision-making in the context of the macroeconomy would be warranted to better understand the broader implications of corporate decisions.

Appendices

A. An Example

In this example, we assume that $f_i(k) = 10\sqrt{k}$ for i=0,1,2, $u(c) = \ln c$, $\delta = 1, \beta = 1/2, d = 100\%, k_0 = 10$. The Euler equation from the consumer's period-1 maximization problem is

$$u'(c_1) = \beta\delta R_2 u'(c_2) \rightarrow \frac{1}{c_1} = \frac{1}{2} R_2 \frac{1}{c_2}. \tag{38}$$

From the firm's period-1 maximization problem, we have

$$R_2 = (1 + \tau_1) f_2'(k_2) \tag{39}$$

By the market clearing condition, we have

$$c_1 = f_1(k_1) - i_1 \text{ and } c_2 = f_2(k_2) \tag{40}$$

Because $d = 100\%$, we have $i_1 = k_2$ and $i_0 = k_1$. From *Eqs.* (38-40), we can get the capital response function:

$$\overline{k}_2(k_1) = \frac{A_1\sqrt{k_1}}{4/(1+\tau_1)+1}. \tag{41}$$

We can solve for the equilibrium by deriving either $\overline{k}_2(k_1)$ or $\overline{b}_1(b_0)$. In this example, the functional form of $\overline{k}_2(k_1)$ is simpler so we get the equilibrium from $\overline{k}_2(k_1)$ rather than $\overline{b}_1(b_0)$. However, we also derive $\overline{b}_1(b_0)$ in *Eq.* (53) below. The Euler equation from the consumer's period-0 maximization problem is

$$-u'(c_0) + \beta\delta u'(c_1)\left(R_1 - \overline{b}_1'(b_0)\right) + \delta u'(c_1)\overline{b}_1'(b_0) = 0 \tag{42}$$

From the firm's period-0 maximization problem, we have

$$R_1 = \frac{1+\tau_0}{1+\tau_1} f_1'(k_1) \tag{43}$$

From Lemmas 1 and 2, we know that $\overline{b}_1'(b_0) = \overline{k}_2'(k_1)$, and the Euler equation of *Eq.* (42) can be written as

$$\begin{aligned} &-u'(f\left(_0(k_0) - i_0\right) + \beta\delta u'(f_1(k_1) - i_1)\left(\frac{1+\tau_0}{1+\tau_1} f_1'(k_1) - \frac{A_1\sqrt{k_1}}{4/\left(1+\tau_1\right)+1}\right) \\ &+\delta u'(f_2(k_2))\frac{A_1\sqrt{k_1}}{4/\left(1+\tau_1\right)+1} = 0 \end{aligned} \tag{44}$$

From *Eq.* (44), we have

$$k_1 = 10\sqrt{k_0}\frac{6+3\tau_1+\tau_1^2+5\tau_0+\tau_0\tau_1}{22+19\tau_1+\tau_1^2+5\tau_0+\tau_0\tau_1}. \tag{45}$$

From Eqs (41) and (45), we can get the equilibrium capital levels; and thus the equilibrium consumption as a function of (τ_0, τ_1). Where $(\tau_0, \tau_1) = (0, 0)$, the equilibrium consumption and utility levels are $(c_0^*, c_1^*, c_2^*) = (22.9984, 23.4939, 24.2352)$ and $\left(U^{(0)}, U^{(1)}, U^{(2)}\right) = (6.3077, 4.7506, 3.1878)$. This paper shows that there is always $(\tau_0, \tau_1) \gg 0$ which Pareto-improves the economy. For example, where $(\tau_0, \tau_1) = (0.5, 0.5)$, the equilibrium consumption and utility levels are $(c_0^*, c_1^*, c_2^*) = (21.9985, 22.5622, 29.0875)$ and $\left(U^{(0)}, U^{(1)}, U^{(2)}\right) = (6.3343, 4.8014, 3.3703)$.

The following is the derivation of $\overline{b}_1(b_0)$ in the example. From the consumer's period-2 maximization problem, we have

$$u'(v_1 + w_1 + R_1 b_0 - b_1) = \beta\delta R_2 u'(v_2 + w_2 + R_2 b_1),$$

and, in turn,

$$\frac{1}{v_1 + w_1 + R_1 b_0 - b_1} = \frac{R_2}{2}\frac{1}{v_2 + w_2 + R_2 b_1}. \tag{46}$$

In equilibrium, we also have

$$R_2 = (1+\tau_1) f_2'(k_2) = (1+\tau_1)\frac{10}{2\sqrt{k_2}}, \tag{47}$$

and

$$v_2 + w_2 = f_2(k_2) - R_2 b_1. \tag{48}$$

Plugging *Eqs.* (47) and (48) into *Eq.* (46), we have

$$\frac{1}{v_1 + w_1 + R_1 b_0 - b_1} = \frac{(1+\tau_1)}{2} \frac{f_2'(k_2)}{f_2(k_2)} = \frac{(1+\tau_1)}{4} \frac{1}{k_2}. \tag{49}$$

By the commodity market clearing conditions, we have

$$c_1 = v_1 + w_1 + R_1 b_0 - b_1 = f_1(k_1) - k_2. \tag{50}$$

From *Eq.* (49) and (50), we have

$$\frac{1}{v_1 + w_1 + R_1 b_0 - b_1} = \frac{(1+\tau_1)}{4} \frac{1}{f_1(k_1) + (1-d)k_1 - (v_1 + w_1 + R_1 b_0 - b_1)},$$

which is equivalent to

$$v_1 + w_1 + R_1 b_0 - b_1 = \frac{4}{(1+\tau_1)} \left\{ f_1(k_1) + (1-d)k_1 - (v_1 + w_1 + R_1 b_0 - b_1) \right\},$$

thus, we have

$$b_1 = \frac{(v_1 + w_1 + R_1 b_0) - \frac{4}{(1+\tau_1)} \left\{ f_1(k_1) - (v_1 + w_1 + R_1 b_0) \right\}}{\frac{4}{(1+\tau_1)} + 1}. \tag{51}$$

By the market clearing condition, we have

$$c_0 = v_0 + w_0 + R_0 b_{-1} - b_0 = f_0(k_0) - k_1. \tag{52}$$

From *Eq.* (51) and (52), we have

$$\begin{aligned} \bar{b}_1(b_0) \quad = \quad & \frac{v_1 + w_1 + R_1 b_0}{4/(1+\tau_1) + 1} \\ & - \frac{4/(1+\tau_1) \left\{ \begin{array}{c} f_1(f_0(k_0) - (v_0 + w_0 + R_0 b_{-1} - b_0)) \\ -(v_1 + w_1 + R_1 b_0) \end{array} \right\}}{4/(1+\tau_1) + 1}. \end{aligned} \tag{53}$$

Differentiating b_1 with b_0, we have

$$\bar{b}_1'(b_0) = \frac{R_1 - 4(f_1'(k_1) - R_1)/(1+\tau_1)}{4/(1+\tau_1) + 1} = \frac{R_1}{4/(1+\tau_1) + 1}, \tag{54}$$

which is the same as $\overline{k}_2'(k_1)$

B. The proof of Lemma 1

The bond market clearing condition is satisfied because we use the same symbol (b_t) for both the consumer's and firm's budget constraints. The commodity market clearing condition is satisfied because the aggregate output in period t, $f_t(k_t)$, is the same as the consumer's total income plus the investment in period $t \in \{0,1\}$, $(v_t + w_t + R_t b_{t-1} - b_t) + I_t$ by the following. The firm's budget constraint is

$$(1-\tau_t)v_t = f_t(k_t) - I_t - w_t - R_1 b_{t-1} + b_t + S_t. \tag{55}$$

Under the revenue-neutral policy, we have $S_t = \tau_t v_t^*$ where the v_t^* is the equilibrium dividend payout in period t. Therefore, *Eq.* (55) in equilibrium can be reduced to

$$v_t = f_t(k_t) - I_t - w_t - R_1 b_{t-1} + b_t, \tag{56}$$

From *Eq.* (56), we have the following equation:

$$(v_t + w_t + R_t b_{t-1} - b_t) + I_t = f_t(k_t). \tag{57}$$

Eq. (57) implies that the commodity market clearing condition is satisfied because $(v_t + w_t + R_t b_{t-1} - b_t)$ is the same as c_t from the consumer's budget constraint.

The remaining proof is to show that for a small value of (τ_0, τ_1) near (0,0), (I) there exists an optimal plan for the firm's investment/dividend/labor levels for any given $R_t \in (0,\infty)$ and $w_t \in (0,\infty)$, (II) there exist a consumer's optimal savings level for any given $R_t \in (0,\infty)$, $w_t \in (0,\infty)$, $v_t \in \mathbb{R}$, and (III) R_t and w_t are finite and strictly positive in equilibrium. We can prove (III) directly from the following first-order conditions:

$$w_t = f_t(k_t) - k_t f_t'(k_t) \quad \text{for t=0,1,2} \tag{58}$$

$$R_1 = \frac{1+\tau_0}{1+\tau_1}\left\{f_1'(k_1) + (1-d)\right\} > 0, \tag{59}$$

$$R_2 = (1+\tau_1)\left\{f_2'(k_2) + (1-d)(1-\chi)\right\} > 0, \tag{60}$$

For the proof of (I), we have shown that the firm maximization problem is well-defined in the *Eqs.* (58-60). For the proof of (II), we can show that the consumer's maximization problem has a unique interior solution with given $R_t \in \mathbb{R}_{++}$, $w_t \in \mathbb{R}_{++}$, $v_t \in \mathbb{R}$:[8] In period

[8] In this paper, to avoid the complication of extra assumptions, we do not restrict the domain of dividends to be strictly positive but we allow negative values of dividends. To restrict the range of the dividend payout requires additional assumptions on production functions. For example, if the

1, the representative consumer chooses b_1 to maximize its period-2 utility function given any (b_0, v_1, w_1, R_1):

$$\max_{b_1} u(c_1) + \beta\delta u(c_2) \tag{61}$$

subject to

$$c_1 = w_1 + v_1 + R_1 b_0 - b_1,$$

$$c_2 = w_2 + v_2 + R_2 b_1.$$

The first-order condition of the maximization problem of *Eq.* (61) is

$$-u'(c_1) + \beta\delta u'(c_2) R_2 = 0. \tag{62}$$

The second-order condition from the maximization problem of *Eq.* (61) is

$$u''(c_1) + \beta\delta u''(c_2) R_2^2 < 0. \tag{63}$$

By the first- and second-order conditions of *Eqs.* (62) and (63), we know that for any value of $b_0 \in (-(w_1 + v_1)/R_1, \infty)$, there exists a unique $b_1 \in \mathbb{R}$ that solves *Eq.* (62). We define $\overline{b}_1(b_0)$, which solves the first-order condition in *Eq.* (62), such that

$$-u'(w_1 + v_1 + R_1 b_0 - \overline{b}_1(b_0)) + \beta\delta u'(w_2 + v_2 + R_2 \overline{b}_1(b_0)) R_2 = 0. \tag{64}$$

Implicitly differentiating *Eq.* (64) with respect to b_0, we have

$$-u''(c_1)\left(R_1 - \overline{b}_1'(b_0)\right) + \beta\delta u''(c_2) R_2^2 \overline{b}_1'(b_0) + \beta\delta u'(c_2) \frac{dR_2}{db_1} \overline{b}_1'(b_0) = 0, \tag{65}$$

Given $(v_1, w_1, R_1, b_0, k_1)$, an increase in b_1 increases the firms investment, i_1; thus, we know that the future capital k_2 increases from the firm's period-1 budget constraint. Therefore, the increase in b_1 affects the future interest rate R_2. Therefore, we have

$$\frac{dR_2}{db_1} = (1 + \tau_1) f_2''(k_2). \tag{66}$$

From *Eqs.* (65) and (66), we have

$$\overline{b}_1'(b_0) = \frac{u''(c_1) R_1}{u''(c_1) + \beta\delta R_2^2 u''(c_2) + \beta\delta u'(c_2)(1 + \tau_1) f_2''(k_2)} > 0. \tag{67}$$

future productivity is extremely high, the firm makes a large investment so the dividend payout is negative.

Plugging $\overline{b}_1(b_0)$ into $U^{(0)}$, we obtain

$$U^{(0)} = u(w_0 + v_0 + R_0 b_{-1} - b_0) + \beta\delta u(w_1 + v_1 + R_1 b_0 - \overline{b}_1(b_0)) + \beta\delta^2 u(w_2 + v_2 + R_2\overline{b}_1(b_0)). \tag{68}$$

By the limiting conditions of utility, such that $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$, we know that the equilibrium capital level b_0 is bounded, i.e., $b_0 \in (w_0+v_0+R_0 b_{-1}, f_0(k_0)+(1-d)k_0)$. This implies that there exists an interior solution b_0 that satisfies the following first- and second-order conditions. The first-order condition is

$$-u'(c_0) + \beta\delta u'(c_1)\left(R_1 - \overline{b}_1'(b_0)\right) + \beta\delta^2 u'(c_2) R_2 \overline{b}_1'(b_0) = 0, \tag{69}$$

and the second-order condition is

$$u''(c_0) + \beta\delta u''(c_1)\left(R_1 - \overline{b}_1'(b_0)\right)^2 + \beta\delta u'(c_1)\left(\frac{1+\tau_0}{1+\tau_1} f_1''(k_1) - \overline{b}_1''(b_0)\right) + \beta\delta^2 u''(c_2)\left(R_2\overline{b}_1'(b_0)\right)^2 + \beta\delta^2 u'(c_2)(1+\tau_1) f_2''(k_2)\left(\overline{b}_1'(b_0)\right)^2 + \beta\delta^2 u'(c_2) R_2 \overline{b}_1''(b_0) \leq 0. \tag{70}$$

C. The proof of Lemma 2

The Euler equation from the period-1 maximization problem is

$$-u'(c_0) + \beta\delta u'(c_1)\left(R_1 - \overline{b}_1'(b_0)\right) + \beta\delta^2 u'(c_2) R_2 \overline{b}_1'(b_0) = 0. \tag{71}$$

where

$$R_1 = \left(\frac{1+\tau_0}{1+\tau_1}\right) \times \{A_1 F_1(K_1, N_1) + (1-d)\}, \tag{72}$$
$$R_2 = (1+\tau_1) \times \{A_2 F_1(K_2, N_2) + (1-d)(1-\chi)\}$$

$$c_0 = f_1(k_0) + (1-d)k_0 - k_1, c_1 = f_1(k_1) + (1-d)k_1 - k_2, \quad c_2 = f_2(k_2) + (1-d)(1-\tau)k_2. \tag{73}$$

Because we have $\overline{b}_1'(b_0) = \overline{k}_2'(k_1)$, the Euler equation in *Eq.* (71) can be expressed as a function of (k_1, k_2) :

$$-u'(c_0) + \beta\delta u'(c_1)\left(R_1 - \overline{k}_2'(k_1)\right) + \beta\delta^2 u'(c_2) R_2 \overline{k}_2'(k_1) = 0. \tag{74}$$

Implicitly differentiating (74) with τ_0, we have

$$\begin{aligned} & u''(c_0)dk_1 + \beta\delta u''(c_1)\left(f_1'(k_1) + (1-d) - \bar{k}_2'(k_1)\right)\left(R_1 - \bar{k}_2'(k_1)\right)dk_1 \\ & +\beta\delta u'(c_1)\left(\frac{1}{1+\tau_1}\right)\left(f_1'(k_1) + (1-d)(1-\chi)\right)d\tau_0 + \beta\delta u'(c_1)\left(\frac{1+\tau_0}{1+\tau_1}f_1''(k_1) - \bar{k}_2''(k_1)\right)dk_1 \\ & +\beta\delta^2 u''(c_2)(1+\tau_1)\left(R_2\bar{k}_2'(k_1)\right)^2 dk_1 + \beta\delta^2 u'(c_2)(1+\tau_1)f_2''(k_2)\bar{k}_2'(k_1)\bar{k}_2'(k_1)dk_1 \\ & +\beta\delta^2 u'(c_2)R_2\bar{k}_2''(k_1)dk_1 = 0. \end{aligned} \tag{75}$$

If $(\tau_0, \tau_1) = (0, 0)$, *Eq.* (75) is equivalent to

$$\begin{aligned} & u''(c_0)dk_1 + \beta\delta u''(c_1)\left(R_1 - \bar{k}_2'(k_1)\right)^2 dk_1 \\ & +\beta\delta u'(c_1)\left(f_1'(k_1) + (1-d)(1-\chi)\right)d\tau_0 + \beta\delta u'(c_1)\left(f_1''(k_1) - \bar{k}_2''(k_1)\right)dk_1 \\ & +\beta\delta^2 u''(c_2)\left(R_2\bar{k}_2'(k_1)\right)^2 dk_1 + \beta\delta^2 u'(c_2)f_2''(k_2)\left(\bar{k}_2'(k_1)\right)^2 dk_1 \\ & +\beta\delta^2 u'(c_2)R_2\bar{k}_2''(k_1)dk_1 = 0. \end{aligned} \tag{76}$$

Remembering the second-order condition in *Eq.* (70), at $(\tau_0, \tau_1) = (0, 0)$, we have

$$\begin{aligned} SOC \; = \; & u''(c_0) + \beta\delta u''(c_1)\left(R_1 - \bar{b}_1'(b_0)\right)^2 + \beta\delta u'(c_1)\left(f''(k_1) - \bar{b}_1''(b_0)\right)dk_1 \\ & +\beta\delta^2 u''(c_2)\left(R_2\bar{b}_1'(b_0)\right)^2 dk_1 + \beta\delta^2 u'(c_2)f_2''(k_2)\left(\bar{b}_1'(b_0)\right)^2 dk_1 \\ & +\beta\delta^2 u'(c_2)R_2\bar{b}_1''(b_0)dk_1 \leq 0. \end{aligned} \tag{77}$$

From *Eqs.* (76) and (77), at $(\tau_0, \tau_1) = (0, 0)$, we have

$$\frac{dk_1}{d\tau_0} = -\frac{\beta\delta u'(c_1)\left(f_1'(k_1) + (1-d)(1-\chi)\right)}{SOC} > 0. \tag{78}$$

Because a period-0 investment subsidy (τ_0) does not change the capital response function $\bar{k}_2(k_1)$, we have the following equality:

$$\frac{dk_2}{d\tau_0} / \frac{dk_1}{d\tau_0} = \bar{k}_2'(k_1). \tag{79}$$

Applying the envelope theorem to the period-0 intertemporal utility, we have

$$\frac{dU^{(0)}}{d\tau_0} = \frac{\partial U^{(0)}}{\partial k_1}\frac{dk_1}{d\tau_0} = -\frac{\partial U^{(0)}}{\partial c_1}\frac{dk_1}{d\tau_0} < 0. \tag{80}$$

The period-1 intertemporal utility is

$$U^{(1)}\left(f_1(k_1)+(1-d)k_1-\overline{k}_2(k_1), f_2(\overline{k}_2(k_1))+(1-d)(1-\chi)\overline{k}_2(k_1)\right). \tag{81}$$

Applying the envelope theorem to *Eq.* (81), we have

$$\frac{dU^{(1)}}{d\tau_0}=\frac{\partial U^{(1)}}{\partial k_1}\frac{dk_1}{d\tau_0}=\frac{\partial U^{(1)}}{\partial c_1}R_2\frac{dk_1}{d\tau_0}>0. \tag{82}$$

The period-2 intertemporal utility is

$$U^{(2)}=u\left(f_2(\overline{k}_2(k_1))+(1-d)(1-\chi)\overline{k}_2(k_1)\right). \tag{83}$$

Applying the envelope theorem to *Eq.* (83), we have

$$\frac{dU^{(2)}}{d\tau_0}=\frac{\partial U^{(2)}}{\partial k_2}\frac{dk_2}{dk_1}\frac{dk_1}{d\tau_0}=\frac{\partial U^{(2)}}{\partial c_2}\overline{k}_2'(k_1)R_2\frac{dk_1}{d\tau_0}>0. \tag{84}$$

D. The proof of Lemma 3

Remembering the Euler equation in *Eq.* (62), we have

$$-u'(c_1)+\beta\delta R_2 u'(c_2)=0, \tag{85}$$

where

$$\begin{aligned} R_2 &= (1+\tau_1)\times\left\{A_2F_1(K_2,N_2)+(1-d)(1-\chi)\right\}, \\ c_1 &= f_1(k_1)+(1-d)k_1-\overline{k}_2(k_1), \\ c_2 &= f_2\left(\overline{k}_2(k_1)\right)+(1-d)\left(1-\chi\right)\overline{k}_2(k_1). \end{aligned} \tag{86}$$

Implicitly differentiating *Eq.* (85) with k_1, we have

$$-u''(c_1)\left(\frac{R_1}{1+\tau_1}-\bar{k}_2'(k_1)\right)+\beta\delta R_2^2u''(c_2)\bar{k}_2'(k_1)+\beta\delta\left(1+\tau_1\right)f_2''(k_2)u'(c_2)\bar{k}_2'(k_1)=0 \tag{87}$$

and, in turn, equivalently,

$$\bar{k}_2'(k_1)=\frac{u''(c_1)\frac{R_1}{1+\tau_1}}{u''(c_1)+\beta\delta R_2^2u''(c_2)+\beta\delta\left(1+\tau_1\right)f_2''(k_2)u'(c_2)}>0, \tag{88}$$

which is the same as $\bar{b}_1'(b_0)$.[9]

Implicitly differentiating *Eq.* (85) with τ_1, we have

$$
\begin{aligned}
& u''(c_1) d\overline{k}_2(k_1) + \beta\delta R_2 u''(c_2)(1+\tau_1)\left(f_2'(k_2) + (1-d)(1-\chi)\right) d\overline{k}_2(k_1) \\
& + \beta\delta u'(c_2)\left(f_2'(k_2) + (1-d)(1-\chi)\right) d\tau_1 = 0,
\end{aligned}
$$

which is equivalent, in turn,

$$
\frac{d\overline{k}_2(k_1)}{d\tau_1} = -\frac{\beta\delta u'(c_2)\left(f_2'(k_2) + (1-d)(1-\chi)\right)}{u''(c_1) + \beta\delta R_2 u''(c_2)(1+\tau_1)\left(f_2'(k_2) + (1-d)(1-\chi)\right)}. \tag{89}
$$

If $\tau_1 = 0$, *Eq.* (89) is reduced to

$$
\frac{d\overline{k}_2(k_1)}{d\tau_1} = -\frac{\beta\delta u'(c_2)\left(f_2'(k_2) + (1-d)(1-\chi)\right)}{u''(c_1) + \beta\delta R_2^2 u''(c_2)}. \tag{90}
$$

The second-order condition from *Eq.* (63), we have

$$
u''(c_1) + \beta\delta u''(c_2) R_2^2 < 0. \tag{91}
$$

From *Eqs.* (90) and (91), where $\tau_1 = 0$, we have

$$
\frac{d\overline{k}_2(k_1)}{d\tau_1} > 0. \tag{92}
$$

Implicitly differentiating $\overline{k}_2(k_1) = k_2$ in terms of τ_1, we have

$$
\frac{d\overline{k}_2(k_1)}{d\tau_1} + \overline{k}_2'(k_1)\frac{dk_1}{d\tau_1} = \frac{dk_2}{d\tau_1}. \tag{93}
$$

From *Eqs.* (92) and (93), we have

$$
\overline{k}_2'(k_1)\frac{dk_1}{d\tau_1} < \frac{dk_2}{d\tau_1}. \tag{94}
$$

In equilibrium, the period-0 intertemporal utility is

$$
U^{(0)}\begin{pmatrix} f_0(k_0) + (1-d)k_0 - k_1, \\ f_1(k_1) + (1-d)k_1 - k_2, \\ f_2(k_2) + (1-d)(1-\tau)k_2 \end{pmatrix}. \tag{95}
$$

[9] This does not imply that $\bar{k}_2(k_1) = \bar{b}_1(b_0)$.

By the envelope theorem, differentiating *Eq.* (95) with τ_1, we have

$$\frac{dU^{(0)}}{d\tau_1} = \frac{\partial U^{(0)}}{\partial k_2}\frac{d\bar{k}_2(k_1^*)}{d\tau_1} = \left\{-\beta\delta u'(c_1) + \beta\delta^2 u'(c_2)R_2\right\}\frac{d\bar{k}_2(k_1^*)}{d\tau_1}. \tag{96}$$

where k_1^* is the equilibrium period-1 capital level where $(\tau_0, \tau_1) = (0, 0)$. From *Eqs.* (62), (96) and (92), we have

$$\frac{dU^{(0)}}{d\tau_1} = (1-\beta)\,\delta u'(c_1)\frac{d\bar{k}_2(k_1^*)}{d\tau_1} > 0. \tag{97}$$

E. Proof of Proposition 1

By the commodity market clearing condition, we can define the intertemporal utilities as functions of capital levels, (k_1, k_2), such that

$$\begin{aligned}\overline{U}^{(0)}(k_1,k_2) &= U^{(0)}(f_0(k_0) + (1-d)k_0 - k_1, \\ &\quad f_1(k_1) + (1-d)k_1 - k_2, f_2(k_2) + (1-d)(1-\chi)k_2),\end{aligned} \tag{98}$$

$$\overline{U}^{(1)}(k_1,k_2) = U^{(1)}(f_1(k_1) + (1-d)k_1 - k_2, f_2(k_2) + (1-d)(1-\chi)k_2), \tag{99}$$

and

$$\overline{U}^{(2)}(k_1,k_2) = U^{(2)}(f_2(k_2) + (1-d)(1-\chi)k_2). \tag{100}$$

We need to show that an increase in both τ_0 and τ_1 can improve all intertemporal utilities. We first check how the intertemporal utilities changes with the level of capital (k_1, k_2). After that, we check how the utility changes with the policy (τ_0, τ_1).

Denote (k_1^*, k_2^*) as the equilibrium capital level where $(\tau_0, \tau_1) = (0, 0)$. At the equilibrium (k_1^*, k_2^*), we have the following inequalities from *Eqs.* (98-100):

$$\frac{\partial \overline{U}^{(2)}}{\partial k_1} = 0, \quad \frac{\partial \overline{U}^{(2)}}{\partial k_2} > 0. \tag{101}$$

$$\frac{\partial \overline{U}^{(1)}}{\partial k_1} > 0, \quad \frac{\partial \overline{U}^{(1)}}{\partial k_2} = 0. \tag{102}$$

$$\frac{\partial \overline{U}^{(0)}}{\partial k_1} < 0, \quad \frac{\partial \overline{U}^{(0)}}{\partial k_2} > 0. \tag{103}$$

The inequalities of *Eq.* (101) can be proven to be true directly from *Eq.* (100). The following are the proofs of the inequalities in *Eqs.* (102-103):

Taking derivative $\overline{U}^{(1)}$ with respect to k_1 at the equilibrium (k_1^*, k_2^*) , we have

$$\frac{\partial \overline{U}^{(1)}}{\partial k_1}|_{(k_1,k_2)=(k_1^*,k_2^*)} = u'(c_1)(f_1'(k_1)+1-d) = u'(c_1)R_1 > 0. \tag{104}$$

Taking the partial derivative of $\overline{U}^{(1)}$ with respect to k_2, we have

$$\begin{aligned}\frac{\partial \overline{U}^{(1)}}{\partial k_2}|_{(k_1,k_2)=(k_1^*,k_2^*)} &= -u(c_1) + \beta\delta(f_2'(k_2)+1-d)u'(c_2) \\ &= -u(c_1) + \beta\delta R_2 u'(c_2),\end{aligned}$$

which is equivalent to the first-order condition in *Eq.* (62). Therefore, we have

$$\frac{\partial \overline{U}^{(1)}}{\partial k_2}|_{(k_1,k_2)=(k_1^*,k_2^*)} = 0. \tag{105}$$

The partial derivative of $\overline{U}^{(0)}$ with respect to k_1 is

$$\begin{aligned}\frac{\partial \overline{U}^{(0)}}{\partial k_1}|_{(k_1,k_2)=(k_1^*,k_2^*)} &= -u'(c_0) + \beta\delta u'(c_1)(f_1'(k_1)+1-d) \qquad (106) \\ &= -u'(c_0) + \beta\delta u'(c_1)R_1.\end{aligned}$$

From *Eqs.* (69) and (106), we have

$$\frac{\partial \overline{U}^{(0)}}{\partial k_1}|_{(k_1,k_2)=(k_1^*,k_2^*)} = \beta\delta\left(u'(c_1) - \delta u'(c_2)R_2\right)\overline{k}_2'(k_1). \tag{107}$$

From *Eq.* (62), we have

$$u'(c_1) = \beta\delta R_2 u'(c_2). \tag{108}$$

From *Eqs.* (107) and (108), we have

$$\frac{\partial \overline{U}^{(0)}}{\partial k_1}|_{(k_1,k_2)=(k_1^*,k_2^*)} < 0.$$

Taking the partial derivative of $\overline{U}^{(0)}$ with respect to k_2 at the equilibrium (k_1^*, k_2^*) , we have

$$\begin{aligned}\frac{\partial \overline{U}^{(0)}}{\partial k_2}|_{(k_1,k_2)=(k_1^*,k_2^*)} &= -\beta\delta u'(c_1) + \beta\delta^2 u'(c_2)(f_2'(k_2)+1-d) \\ &= \beta\delta\left(-u'(c_1) + \delta u'(c_2)R_2\right). \qquad (109)\end{aligned}$$

Remembering the first-order condition in *Eq.* (62), we have

$$-u'(c_1) + \beta\delta u'(c_2)R_2 = 0. \tag{110}$$

From *Eqs.* (109) and (110), we have

$$\frac{\partial \overline{U}^{(0)}}{\partial k_2}|_{(k_1,k_2)=(k_1^*,k_2^*)} > 0.$$

From *Eqs.* (101) and (102), we know that any marginal increase in k_1 would increase $U^{(1)}$ and $U^{(2)}$. However, from *Eqs.* (106) and (109), small increases in capital $(\Delta k_1, \Delta k_2)$ should satisfy the following inequality to induce a $U^{(0)}$ increase:

$$\frac{\Delta k_2}{\Delta k_1} > -\frac{-u'(c_0) + \beta\delta R_1 u'(c_1)}{\beta\delta\left(-u'(c_1) + \delta R_2 u'(c_2)\right)} > 0. \tag{111}$$

Remembering *Eq.* (69), we have

$$-u'(c_0) + \beta\delta u'(c_1)\left(R_1 - \overline{b}_1'(b_0)\right) + \beta\delta^2 u'(c_2)R_2\overline{b}_1'(b_0) = 0 \tag{112}$$

From *Eq.* (112), we know that the inequality in *Eq.* (111) is equivalent to

$$\frac{\Delta k_2}{\Delta k_1} > \overline{b}_1'(b_0), \tag{113}$$

which is also equivalent to $\Delta k_2/\Delta k_1 > \overline{k}_2'(k_1)$.

Therefore, if a small increase in (τ_0, τ_1) from $(0,0)$ to $(\Delta\tau_0, \Delta\tau_1)$ increases (k_1, k_2) in the same way as in *Eq.* (113), the policy would be Pareto-improving. From Lemma 2, we have

$$\frac{dk_2}{d\tau_0}\Delta\tau_0 = \overline{k}_2'\left(k_1\right)\frac{dk_1}{d\tau_0}\Delta\tau_0. \tag{114}$$

From Lemma 3, we have

$$\frac{dk_2}{d\tau_1}\Delta\tau_1 > \overline{k}_2'\left(k_1\right)\frac{dk_1}{d\tau_1}\Delta\tau_1. \tag{115}$$

From *Eqs.* (114) and (115), we have

$$\frac{dk_2}{d\tau_0}\Delta\tau_0 + \frac{dk_2}{d\tau_1}\Delta\tau_1 > \overline{k}_2'\left(k_1\right)\left(\frac{dk_1}{d\tau_0}\Delta\tau_0 + \frac{dk_1}{d\tau_1}\Delta\tau_1\right), \tag{116}$$

which is equivalent to the inequality in *Eq.* (113). Therefore, there always exists a Pareto-improving subsidy policy $(\tau_1, \tau_2) \gg 0$.

F. Proof of Proposition 2

For the existence of equilibrium, we can directly use the result in Lemma 1. Then, we can prove the Proposition with the same logic (in a reverse way) of the proof of Proposition E. In proposition E, the main forces that increase consumer's savings are higher interest rates, which is led by higher dividend taxes. However, the income taxes decrease the real interest rates as shown below:

$$R_2 = A_2 F_1(K_2, N_2)\,(1-\theta_2) + (1-d)(1-\chi) \text{ and}$$

$$R_1 = A_1 F_1(K_1, N_1)\,(1-\theta_1) + (1-d).$$

Applying the reverse logic in Proposition 1, we can show that an increase in (θ_1, θ_2) moves the equilibrium capital level into the Pareto-inferior region.

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