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Optimal consumption taxes on durable and nondurable goods purchases with present bias

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Abstract

This paper investigates an optimal tax plan in a general-equilibrium model with nondurable-durable goods under quasi-hyperbolic discounting preferences. The optimal tax policy in the model should be designed in a way that increases the ratio of durable-nondurable purchases as well as the capita-to-output ratio. Thus, homogenous consumption tax that does not distinguish nondurable and durable goods cannot maximize welfare. The quantitative analysis shows that the welfare lifetime gain from the optimal consumption tax is equivalent to gaining an additional 6.9% of period-0 nondurable consumption goods.

Keywords: Consumption taxes; Durable goods; Present bias; Hyperbolic discounting; Steady state analysis

JEL classification: E03; E21; H21

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1 Introduction

Empirical evidence suggests that consumers' discounting functions are approximately hyperbolic rather than time consistent (Thaler 1981; Ainslie 1992; Loewenstein and Prelec 1992). Based on this evidence, Strotz (1956), Phelps and Pollak (1968), and Laibson (1996, 1997) have developed a present biased preferences model: the quasi-hyperbolic discounting model. With time-inconsistent preferences, a sequence of selves has ordinally different lifetime utility, which results in non-optimal Nash equilibria. Therefore, governments would have incentives to intervene to affect consumers' decisions and improve welfare. Numerous research studies show that capital subsidies or consumption taxes can curb consumers' bias and thus improve welfare.[1] However, all previous models with optimal taxation under present bias assume that all consumption goods are nondurable. This paper constructs a nondurable-durable-savings model and derives the optimal consumption tax rates. The optimal tax plan derived in this paper is considerably different from the previous understanding based on homogeneous perishable consumption goods. To the best of my knowledge, this is the first paper to investigate the optimal tax on durable goods together with nondurable goods under present-biased preferences.[2]

As Laibson (1997) indicates, durable goods can be used as a commitment tool under hyperbolic discounting because it can contribute to future utility (i.e., future reward). However, it also provides an immediate reward that is similar to nondurable consumption goods. Therefore, it is difficult to determine whether the durable goods are undervalued (due to immediate reward) or overvalued (due to the commitment role) and, thus, to determine how to design the optimal consumption taxes on durable goods. This paper shows that the optimal consumption tax rates for both durable and nondurable goods should be different. First, both tax rates should be strictly positive, which means that compared to the value of savings, consumers overvalue both durable and nondurable goods. Second, the tax rate for durable goods should be smaller than that for nondurable goods, which implies that the consumer overvalues nondurable goods more than durable goods. Third, the tax rate for durable goods purchases

[1] For capital subsidies, see Krusell, Kurus? and Smith 2010, Pavoni and Yazici 2017. For consumption taxes, see Laibson 1996, O'Donoghue and Rabin 2006, and Kang 2019. For corporate tax policy to resolve the underinvestment problem caused by consumers' present bias, see Kang 2020.

[2] A few papers investigate the role of durable goods but not in an optimal tax context. To the best of my knowledge, the only research paper that incorporates durable goods using the hyperbolic consumption model is Nocke and Peitz (2003), in which optimal tax policy is not the topic. They show that under hyperbolic discounting time preferences, durable goods can be used as a commitment device in the secondary market.

depends on future interest rates because current purchases on durable goods affect future utility as well as current utility.

To derive the optimal taxation, this paper uses commitment preferences (i.e., normative preferences) that have been actively used for policy implications (e.g., O'Donoghue and Rabin 1999, Diamond and Kozegi 2003, Bisin, Lizzeri, and Yariv 2015). Specifically, O'Donoghue and Rabin (1999, 2015) proposed a long-term perspective welfare criterion, which defines time-t as the self's normative preference in the prior time period (i.e., period t-1). Under quasi-hyperbolic time preferences, this long-term preference necessarily follows exponential discounting time preferences. Separately, Krusell, Kurusçu and Smith (2010) apply Gul and Persendorfer's (2001, 2004) axiomatic approach for the self-control problem to the typical macroeconomic model and show that commitment utility follows the exponential discounting time preferences when the consumer has fully succumbed to temptation. Caliendo and Findley (2019) also provide an additional justification for commitment preferences by showing that the time-zero consumption plan (i.e., commitment allocation) is Pareto-superior to the equilibrium consumption plan associated with hyperbolic discounting time preferences.[3]

The optimal consumption taxation should correct consumers' two main suboptimal decisions. In each period, the current self faces her own present bias of that period. From a paternalistic perspective (i.e., based on normative preferences), this current bias should be corrected. In addition, when the current self makes consumption-savings decisions, she needs to consider the present bias of her future self. This second effect, called a time-inconsistent effect, additionally distorts consumption-savings decisions together with the first effect. The optimal tax policy in this paper can resolve these two problems and, thus, maximize welfare.

This paper also provides quantitative analysis on welfare gains from the policy. To quantify the welfare impact of the optimal tax policy, this paper derives steady state equilibrium without a tax policy. At the steady state, we assume that the government implements the optimal policy in period 1. We compare two economies: one is an economy with an optimal tax policy so the equilibrium converges to the new steady state. The other is an economy without an optimal tax policy so it

[3] The two major welfare criteria for quasi-hyperbolic discounting models are the Pareto criterion that takes into account all periods' intertemporal utility and the normative criterion that considers lifetime utility in fictitious period 0. Caliendo and Findley (2019) show that the policy that maximizes normative utility also improves all intertemporal utility. Conversely, Kang (2015) and Kang and Wang (2019) show that efficiency by the Pareto criterion implies efficiency by the normative criterion.

maintains steady state equilibrium. We calculate the welfare gain in terms of period-1 nondurable goods. The welfare gain is equivalent to 6.9% of period-1 nondurable consumption goods with reasonable parameter choices.[4] The numerical analysis also shows that the optimal consumption tax rates on nondurable goods and durable goods are about 42% and 6%, respectively. The large gap between the two tax rates implies that uniform consumption tax, which does not distinguish between durable and nondurable goods, is not an efficient way to maximize welfare in the economy with present-biased preferences.

The remainder of the paper is organized as follows. Section 2 introduces a macroeconomic model in which a present-bias consumer consumes both durable and nondurable goods and a firm produces both types of goods. Section 3 presents optimal consumption tax rates. Section 4 introduces a steady-state analysis of the hyperbolic economy with consumption tax policies. Section 5 concludes. All the proofs of propositions and lemmas are in the Appendices.

2 Model

This section introduces a T-period general-equilibrium macroeconomic model under quasi-hyperbolic discounting, which can be extended into an infinite-period model under the government's optimal tax policy. In this model, a present-biased representative consumer consumes nondurable and durable goods and a representative firm produces durable goods, nondurable goods, and capital.

2.1 Period utility

In each period, the consumer receives the period utility from an index of consumption services of durable and non-durable goods. The consumer period utility is

$$U(c_t, d_t) = u(c_t) + v(d_t),$$

where c_t denotes consumption of non-durable goods, and d_t denotes services from the stock of durable goods at the end of period t. Period utility $U(c_t, d_t)$ is twice-continuously differentiable, strictly increasing, strictly concave, $\lim_{c_t \to 0} U_1(c_t, d_t) = \infty$, $\lim_{d_t \to 0} U_1(c_t, d_t) = \infty$, $\lim_{c_t \to \infty} U_2(c_t, d_t) = 0$, and $\lim_{d_t \to \infty} U_2(c_t, d_t) = 0$.[5]

[4] In the numerical analysis, β=0.7 and δ=0.981, which are commonly used in the literature. See Laibson, Repetto, and Tobacman (2007).

[5] This assumption of additively separability of the period utility is to ensure the existence of equilibrium. Even if the utility is not separable, the main results of the optimal taxation in Proposition

The budget constraint in period t is

$$c_t + p_t\left(d_t - (1-\eta)d_{t-1}\right) + k_{t+1} = R_t k_t + w_t, \tag{1}$$

where k_{t+1} is the capital asset holding in period $t+1$, R_t is the gross real interest rate in period t and w_t is the real wage in period t, η is the durable good depreciation rate, and p_t is the relative price of the durable goods. In period t, the consumer buys the durable good for the amount of $(d_t - (1-\eta)d_{t-1})$. In period 0, the consumer is endowed with k_0 and d_{-1}. In the last period, which is period T, the consumer can liquidate the durable goods with the proportional liquidation cost $\varphi \in (0,1)$. Therefore, the period-T budget constraint should be

$$c_T + p_T\left(d_T - (1-\eta)d_{T-1}\right) = R_t k_t + w_t + (1-\varphi)d_T. \tag{2}$$

2.2 Quasi-hyperbolic discounting

This paper assumes that the representative consumer is present biased. Specifically, we use the popular β-δ model that by Strotz (1956), Pollak (1968), Phelps and Pollak (1968), and Laibson (1997). There are T periods in the economy. This T-period model can be extended into an infinite-period model assuming $T \to \infty$. Under quasi-hyperbolic discounting, given the budget constraints in Eq. (1), period-t self maximizes to the following intertemporal utility:

$$U(c_t, d_t) + \beta\delta\left\{U(c_{t+1}, d_{t+1}) + \delta U(c_{t+2}, d_{t+2}) + ... + \delta^{T-t-1}U(c_T, d_T)\right\}. \tag{3}$$

where $\beta \in (0,1)$ represents the hyperbolic discounting factor and $\delta \in (0,1)$ represents a long-term discounting factor.

2.3 The production function

Let f_t be the standard neoclassical aggregate production function at period t and let d be the capital depreciation rate. The firm can produce nondurable goods,

1 still holds as long as equilibrium exists. When the value of the cross derivative U_{12} is negative, the consumer can choose negative consumption on the durable goods purchase, in which case equilibrium does not exist.

durable goods, and capital, thus the commodity market clearing condition is

$$f\left(k_t\right)+(1-d)k_t=c_t+d_t+k_{t+1}. \tag{4}$$

where k_t and k_{t+1} are the capital levels in periods t and $t+1$, respectively. Eq. (4) implies that the marginal rate of substitution between c_t and d_t is one; thus, in equilibrium, the relative price of the durable good (p_t) in Eq. (1) should be one as well.[6]

The wages and interest rate are determined by the aggregate savings behavior of consumers. Specifically, the real wage and real gross interest rate in period t is

$$R_t \equiv R_t(\overline{k}_t)=1-d+f_t'\left(\overline{k}_t\right) \tag{5}$$

$$w_t \equiv w_t(\overline{k}_t)=f\left(\overline{k}_t\right)-f_t'\left(\overline{k}_t\right)\overline{k}_t \tag{6}$$

where $\overline{k}_t$ represents an aggregate capital level in period t. We assume that the aggregate production function is

$$f_t(k_t)=z_t k_t^{\alpha}, \tag{7}$$

where z_t is the period-t total factor productivity and α is the national capital share.

2.4 Efficient allocations

In this paper, efficient allocations are given by the solution to a social planner's consumption-saving problem where the planner discounts exponentially with discount factor δ. Specifically, this paper assumes that there is a beneficial government who evaluates consumers' welfare based on normative preferences. The vast majority of the literature dealing with economic policies under present bias uses these normative preferences for policy evaluations. For example, O'Donoghue and Rabin (1996, 2015) state that the welfare in period t should be evaluated with prior period $t-1$ and, thus, the normative welfare function in each period should not be affected by the hyperbolic discounting factor α. Second, Krusell, Kurusçu and Smith (2010) apply the Gul-Persendorfer's axiomatic approach for the self-control problem to a typical macroeconomic model. They show that when the consumer fully succumbs to temptation, the consumer decision is determined by the temptation utility (which is equivalent to the quasi-hyperbolic discounting utility), but the true welfare is determined by the

[6]Even though the marginal rate of substitution is not one but has some value $h \neq 1$, the main result of this paper is invariant because replacing the utility $v(d_t)$ with $v(d_t/h)$, we have the same equilibrium prices as in the economy with $v(d_t)$ and the marginal rate of substitution being one.

commitment utility (with discounting factor δ).[7]

Based on the normative welfare function (i.e., $\beta = 1$), the following Euler equations and the first-order condition characterize the efficient allocation:[8]

$$U_1(c_t, d_t) = \delta R\left(\overline{k}_{t+1}\right) A_t, \tag{8}$$

$$U_2(c_t, d_t) + \delta(1-\eta)A_t = \delta R(\overline{k}_{t+1})A_t \tag{9}$$

and

$$U_1(c_t, d_t) = U_2(c_t, d_t) + \delta(1-\eta)A_t. \tag{10}$$

where

$$A_t = U_1(c_{t+1}, d_{t+1}) = U_2(c_{t+1}, d_{t+1}) + \delta R(\overline{k}_{t+2})(1-\eta)A_{t+1}. \tag{11}$$

Eqs. (8) and (9) are the first-order conditions in terms of period-t nondurable goods and durable goods, respectively. Eq. (10) implies that at each period, the marginal period-utility of durable goods is the same as that of nondurable goods.

From Eqs. (8-10), efficient allocations can be characterized by two Euler equations:

$$U_1(c_t, d_t) = \delta R\left(\overline{k}_{t+1}\right) U_1(c_{t+1}, d_{t+1}). \tag{12}$$

and

$$U_2(c_t, d_t) = \delta\left\{R\left(\overline{k}_{t+1}\right) - (1-\eta)\right\} U_1(c_{t+1}, d_{t+1}). \tag{13}$$

3 Consumption taxation

The previous section derives Euler equations from normative preferences. This section introduces consumption taxes that can achieve the first-best allocations. The main result of this section shows that the consumption tax rate for nondurable goods should be higher than that for durable goods to maximize welfare.

[7] Much research has used normative preferences in policy evaluations. See O'Donoghue and Rabin (1999, 2001, 2003, 2006), DellaVigna and Malmendier (2004), Diamond and Koszegi (2004), Guo and Krause (2015), Pavoni and Yazici (2017).

[8] In this time-consistent case ($\beta = 1$), we do not need the recursive forms of Euler equations using A_t and A_{t+1}. However, for a direct comparison with the Euler equations with a time-inconsistent case ($\beta < 1$) in the next section, we write the Euler equations in a recursive way, as shown in Eqs. (8-11).

3.1 Proportional consumption taxes

We assume that the government can implement different tax rates on durable and nondurable goods. Specifically, with the proportional consumption taxes, the budget constraint should be

$$(1+\tau_{n,t})\,c_t + (1+\tau_{d,t})\,p_t\,(d_t - (1-\eta)d_{t-1}) + k_{t+1} = R_t k_t + w_t + S_t. \qquad (14)$$

where $\tau_{n,t}$ and $\tau_{d,t}$ represent tax rates of nondurable and durable consumption goods at period t, respectively. S_t is the lump-sum subsidy. Under the revenue neutral policy, we have

$$S_t = \tau_{n,t}c_t^* + \tau_{d,t}p_t\left(d_t^* - (1-\eta)d_{t-1}^*\right). \qquad (15)$$

where c_t^* and d_t^* is the equilibrium nondurable and durable goods purchases at period t. The government runs a balanced budget as shown in Eq. (14) but this assumption is not restrictive. A Ricardian equivalence states that given the consumption taxes, the government deficits and surpluses financed by lump-sum taxes would have no effect on equilibrium allocation. Therefore, instead of the assumption of the revenue-neutral policy, by assuming that the government can freely access the financial markets, we can get the same equilibrium outcomes.

The equilibrium of the economy is characterized by the consumer's maximization problems in *Eqs.* (3) and (14) given $(R_t, w_t)_{t=0}^2$; the firm's maximization problems implied in *Eqs.* (5-6), given $(R_t, w_t)_{t=0}^2$; the government budget is satisfied and the market clears as shown in Eq. (15).

3.2 Optimal consumption taxes

This subsection derives the Euler equations where the consumer is affected by the time-inconsistent problem (i.e., $\beta < 1$) and the government implements the linear consumption taxes. Under the time-inconsistency preferences, the consumers' decisions are affected by two types of myopia. First, the current consumer makes consumption-savings decisions based on her current lifetime utility, which is affected by the current myopic parameter (β). Second, the current consumer also considers that her future self will make decisions based on the future myopic parameters (β). Due to the second effect, the Euler equation should be derived in a recursive way to consider all future myopic decisions.

The following Euler equations and the first-order condition characterize the competitive equilibrium:

$$U_1(c_t, d_t) = (1 + \tau_{n,t})\, \beta\delta R\left(\overline{k}_{t+1}\right) A_t \tag{16}$$

and

$$U_2(c_t, d_t) + \beta\delta(1-\eta)A_t = (1 + \tau_{d,t})\, \beta\delta R(\overline{k}_{t+1})A_t \tag{17}$$

and

$$\frac{U_1(c_t, d_t)}{U_2(c_t, d_t) + \beta\delta(1-\eta)A_t} = \frac{(1+\tau_{n,t})}{(1+\tau_{d,t})} \tag{18}$$

In the time-*consistent* model, A_t can be simply expressed as the period t+1 utility function as shown in Eqs. (12-13). However, under hyperbolic discounting, A_t is a function of all future periods' utility functions as indicated in Laibson (1997). For the current self, the future self's lifetime utility is not consistent with the current period lifetime utility. Consequently, even if the current self has one unit of savings, from the current self's perspective, the one unit will not be used "effectively" by the future self. Therefore, in the Euler equations of Eqs. (16-17), A_t is not identical to $U_1(c_{t+1}, d_{t+1})$.[9]

Specifically, A_t is defined as recursive as follows:

$$\begin{aligned} A_t \;=\; & \lambda_{t+1}\left\{e_{t+1}U_1(c_{t+1}, d_{t+1}) + (1-e_{t+1})\left(U_2(c_t, d_t) + \delta(1-\eta)A_{t+1}\right)\right\} \\ & + (1-\lambda_{t+1})\,\delta R\left(\overline{k}_{t+2}\right) A_{t+1} \end{aligned} \tag{19}$$

where λ_{t+1} is the consumption rate in period $t+1$ and e_{t+1} is the ratio of the expenditure of nondurable consumption to the total consumption in period $t+1$. Both λ_{t+1} and e_{t+1} are functions of the tax policies so there is no simple way to derive their exact functional form from the Euler equations.[10]

This paper shows that there are future consumption tax policies that eliminate the future self-control problem, as shown in Proposition 1. Then, with the future tax policies, the Euler equation in the current period (period t) should be $U_1(c_t, d_t) = (1+\tau_{n,t})\, R\left(\overline{k}_{t+1}\right) \beta\delta U_1(c_{t+1}, d_{t+1})$, which implies that $A_t = U_1(c_{t+1}, d_{t+1})$.

Before we suggest the specific optimal tax rates, we first show that if the future self behaves in a time-consistent way (i.e., $\beta = 1$), A_t can be expressed as the marginal

[9] Under logarithm utility function, Laibson (1997) shows that A_t can be a linear function of $U_1(c_{t+1}, d_{t+1})$ so we have $U_1(c_t, d_t) = \beta\delta R\left(\overline{k}_{t+1}\right)\psi U_1(c_{t+1}, d_{t+1})$ where $\psi \in (1, 1/\beta)$ is a constant. However, in the model with durable goods, ψ cannot be expressed as a constant.

[10] However, with logarithm utility, the savings rate is not affected by the income level, so we can solve for them in a the steady state, which will be shown in the next section.

utility of period t+1 utility in the Euler equations of Eqs. (16-17) in the following lemma:

Lemma 1 *If the Euler Equations in period* $s+1$ *for* $s = t\,, t+1, ..., T$ *are*

$$U_1(c_{s+1}, d_{t+1}) = \delta R\left(\overline{k}_{s+2}\right) U_1(c_{s+2}, d_{s+2}) \tag{20}$$

and

$$U_2(c_{s+1}, d_{s+1}) + \delta(1-\eta) U_1(c_{s+2}, d_{s+2}) = \delta R(\overline{k}_{s+2}) U_1(c_{s+2}, d_{s+2}), \tag{21}$$

then, the Euler equation in period t *with the period-t tax policy should be*

$$U_1(c_t, d_t) = (1+\tau_{n,t})\,\beta\delta R\left(\overline{k}_{t+1}\right) U_1(c_{t+1}, d_{t+1})$$

and

$$U_2(c_t, d_t) + \beta\delta(1-\eta) U_1(c_{t+1}, d_{t+1}) = (1+\tau_{d,t})\,\beta\delta R(\overline{k}_{t+1}) U_1(c_{t+1}, d_{t+1})$$

Proof: See Appendix A.

Lemma 1 shows that if future self behaves in a time-consistent way with the exponential discounting factor, δ, the current Euler equations can be expressed in a non-recursive way (i.e., A_t is a linear function of $U_1(c_{t+1}, d_{t+1})$). The remaining proof shows that a tax policy makes all future and current selves behave in a time-consistent way. Then, such a policy would be the optimal paternalistic tax policy to maximize normative welfare. The following proposition indicates the optimal consumption tax rates in a T-period model:

Proposition 2 *In the T-period model, the optimal consumption tax rates for non-durable and durable purchases are*

$$\tau_{n,t} = \frac{1}{\beta} - 1 \tag{22}$$

and

$$\tau_{d,t} = \frac{R\left(\overline{k}_{t+1}\right) - (1-\beta)(1-\eta)}{\beta R\left(\overline{k}_{t+1}\right)} - 1, \tag{23}$$

for $t = 0, 1, ..., T-1$. *The last period's (i.e., period T), the tax rates should satisfy*

$$\tau_{n,T} = \tau_{d,T}.$$

Proof: See Appendix B.

Proposition 1 provides three important results. First, the consumption tax rates for both durable and nondurable goods should be positive. This means that the consumer overvalues both nondurable and durable goods while she undervalues savings. Second, the tax rate on nondurable goods is smaller than that on durable goods for all periods. This implies that the consumer overvalues nondurable goods more than durable goods. Third, the durable tax rate in period t depends on the future interest rates because the present value of the durable goods depends on the future self's utility as well as the current self's utility.

This model can be extended to an infinite model where $T \to \infty$. Even in the infinite-period model, the optimal tax policy should satisfy Eqs. (22-23). In the next section, in the infinite period model, we derive the steady-state equilibrium with the tax policy and quantify the welfare gain from the policy.

4 The steady-state welfare analysis

This section analyzes the steady state equilibrium of the economy with/without the tax policy. This steady state analysis is necessary to quantify the welfare gain from the optimal tax policy. We assume that in period 0, the economy is at the steady state without the tax policy. In period 1, the optimal tax policy is applied, and we calculate the welfare gain of the tax policy in terms of nondurable consumption goods.

4.1 Steady state equilibrium without the tax policy

For a numerical analysis, we assume that period-utility is additively separable and log-linear as follows:

$$U(c_t, d_t) = u(c_t) + v(d_t) = (1-b)\ln c_t + b \ln d_t, \tag{24}$$

where b is the share of durable goods in the composite consumption index. Then, as described in the previous section, the equilibrium without the tax policy is characterized by the following Euler equations and first-order conditions:

$$u'(c_t) = \beta\delta R_{t+1} A_t, \tag{25}$$

$$u'(c_{t+1}) = \beta\delta R_{t+2} A_{t+1}, \text{ and} \tag{26}$$

$$u'(c_t) = v'(d_t) + \beta\delta (1 - \eta) A_t, \tag{27}$$

where

$$\begin{aligned} A_t &= \lambda_t \{e_t u'(c_{t+1}) + (1 - e_t) (v'(d_{t+1}) + \delta (1 - \eta) A_{t+1})\} \\ &\quad + (1 - \lambda_t) \delta R_{t+2} A_{t+1}. \end{aligned} \tag{28}$$

Under the homothetic preferences, consumption in each period can be characterized by the following rule:

$$c_t + d_t - (1 - \eta)d_t = \lambda_t W_t, \tag{29}$$

where W_t is the sum of the financial assets, the discounted value of future labor income, and λ_t is the consumption rate.[11] Then, the nondurable/durable purchases in period t can be characterized as

$$c_t = \lambda_t e_t W_t, \tag{30}$$

$$d_t - (1 - \eta)d_{t-1} = \lambda_t (1 - e_t) W_t. \tag{31}$$

and

$$c_{t+1} = (1 - \lambda_t) W_t R_{t+1} \lambda_{t+1} e_{t+1}, \tag{32}$$

where e_t is the share of the durable goods expenditure. In a steady state, R_t, λ_t and e_t are constant over time such as $R^* = R_t, \lambda^* = \lambda_t$, and $e^* = e_t$.

The total factor productivity (z_t) is assumed to grow exogenously at the rate g_z. Therefore, in a steady state, capital and output must grow at rate $g_z/(1 - \alpha) \equiv g$ and we have

$$d_{t+1} = d_t \exp(g),\ c_{t+1} = c_t \exp(g) \tag{33}$$

and

$$R^*(1 - \lambda^*) = \exp(g). \tag{34}$$

[11] This linear consumption approach is initially proposed by Laibson (1996) (see *Eq.* (8) on Page 11). This paper proves that consumption is linearly proportional to the life-time wealth under the log-linear utility as in Eq. (24).

From Eqs. (25), (26) and (28), we have

$$\frac{u'(c_t)}{\beta\delta R_{t+1}} = \lambda_t \left\{ e_t u'(c_{t+1}) + (1-e_t)\left(v'(d_{t+1}) + \delta(1-\eta)\frac{u'(c_{t+1})}{\beta\delta R_{t+2}} \right) \right\} + (1-\lambda_t)\frac{u'(c_{t+1})}{\beta} \tag{35}$$

From Eqs. (27) and (25), we have

$$u'(c_t) = v'(d_t) + (1-\eta)\frac{u'(c_t)}{R_{t+1}} \tag{36}$$

Plugging c_t, c_{t+1}, d_t, d_{t+1} in Eqs. (30-33) into (34-36) and replacing (R_t, λ_t, e_t) with (R^*, λ^*, e^*), we can solve for (R^*, λ^*, e^*) from the three equations in Eqs. (34-36).[12]

4.2 Parameter choices and calibration

For the same parameter choices as Laibson (1997), we set $\alpha = 0.36, d = 0.08$, and $g = 0.02$. Based on the empirical data (see Angeletos et al. 2001 and Laibson, Repetto, and Tobacman 2007), the value of the annual discounting factor (β) for a typical household is estimated at around 0.6 to 0.7. We choose $\beta = 0.7$ and $\delta = 0.981$ in this paper.

In this model, we also need to choose the parameters related to utility functions of the durable good services and the depreciation rate for durable goods. The durable good depreciation rate for vehicles is around 15% and for housing is around 3%. In this model, we assume that $\eta = 10\%$. The share of durable consumption of total private consumption spending is around 10-15%. We assume that the durable consumption share is 13%. Therefore, in the steady state, we have

$$\frac{d_t^* - (1-\eta)d_{t-1}^*}{c_t^* + d_t^* - (1-\eta)d_{t-1}^*} = 0.13 = 1 - e^*. \tag{37}$$

Because the share of durable-good expenditure (e^*) strictly decreases with an increase in parameter b in the period-utility in Eq. (24), we can calibrate b from Eq. (37). Based on the calibration we have $b = 0.15$. With the parameter choices above, we have the following steady state equilibrium:

$$R^* = 1.0467, e^* = 0.870815, \text{ and } \lambda^* = 0.0253165. \tag{38}$$

[12] In Eqs. (35-36), W_t is cancelled so we do not need to solve for W_t.

From the Cobb-Douglas production function of $y_t = z_t k_t^\alpha$, the real gross interest rate R^* is expressed by

$$R^* = \alpha/k^* - d + 1. \tag{39}$$

where k^* is the steady state per-capita capital level. From Eq. (**??**), we can also derive k^* from R^*. From the steady state variables in Eq. (38), we can derive the capital-to-output ratio:

$$\frac{Y_t^*}{K_t^*} = \frac{1}{k^*} = \frac{\alpha}{R^* - (1-d)} = \frac{0.36}{1.0467 - (1 - 0.08)} = 2.8414 \tag{40}$$

We can also derive the durable-stock to nondurable-consumption ratio in the steady state:

$$\begin{aligned} \frac{d_t^*}{c_t^*} &= \frac{1-e^*}{e^*} \times \frac{\exp(g)}{\exp(g) - (1-\eta)} \\ &= \frac{1 - 0.870815}{0.870815} \times \frac{\exp(0.02)}{\exp(0.02) - (1 - 0.1)} = 1.2591 \end{aligned} \tag{41}$$

Laibson (1997) indicates that the value of 2.8414 for the capital-to-output ratio is lower than the optimal level. This paper indicates that in an economy with nondurable and durable goods, neither the capital-to-output ratio nor nondurable-to-durable ratio reach the optimal level. The optimal tax policy proposed this paper will increase both ratios effectively.

4.3 Welfare gain from the policy

Assume that in period 0, the economy is at the steady-state without the tax policy. The government implements the optimal tax policy in period 1. The numerical approach for finding the convergence path includes the following steps. First, we guess the equilibrium consumption c_1. Because d_1 is affected by the future interest rate R_2, we cannot directly get d_1 from c_1. Therefore, we need to solve for (d_1, K_1, Y_2, R_2) from the following for equations:

$$\text{Budget constraint: } k_2 = y_1 + (1-d)k_1 - c_1 - d_1 + (1-\eta)d_0,$$

$$\text{Production function: } y_2 = A_1 \exp(g(1-a))k_2^a,$$

$$\text{Capital demand: } R_2 = (1-d) + a\frac{1}{k_2},$$

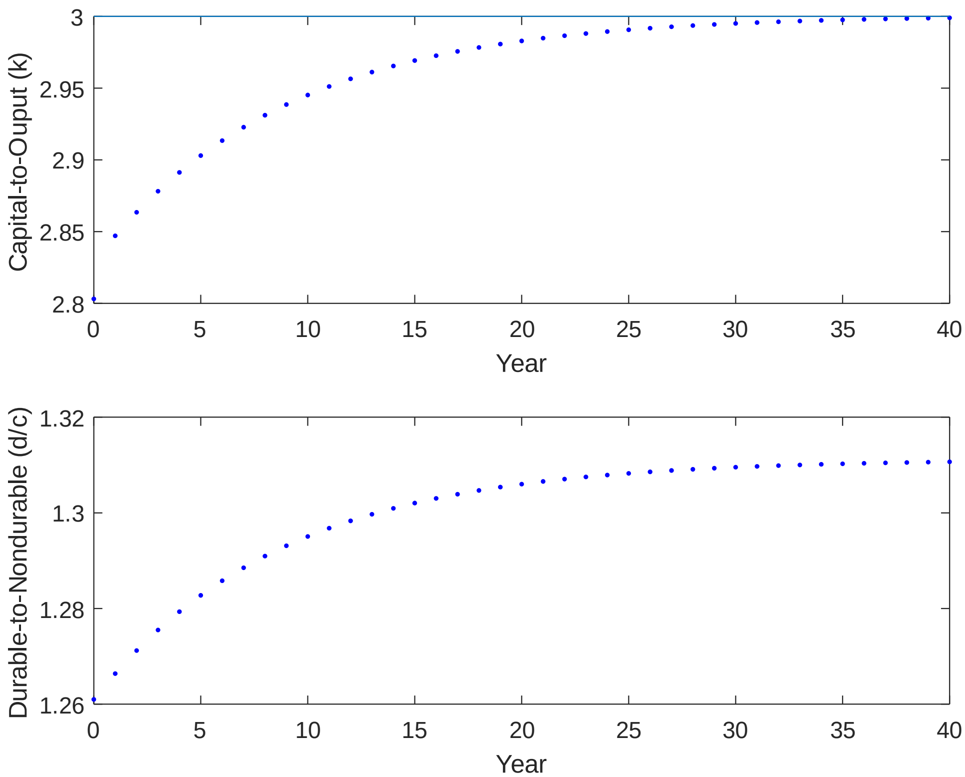

Figure 1: In period 0, the economy with $\alpha = 0.36, d = 0.08, g = 0.02, \beta = 0.7, \delta = 0.981, b = 0.15, \eta = 0.1$ is at the steady state without the tax policy. The optimal policy is applied in period 1. Then, there will be increases in both the capital-to-ouput and durable-to-nondurable ratios from period 1. Over time, the equilibrium converges to the new steady state ($k^* = 3$ and $d_t^*/c_t^* = 1.31$).

and

$$\text{Euler equation: } c_1 \frac{R_2}{R_2 - (1-\eta)} \frac{b}{1-b} = d_1.$$

Then, by the Euler equation of *Eq.* (25) with an optimal tax policy (i.e., $A_1 = u'(c_2)$), c_2 can be derived. In this way, we can derive a sequence of $\{c_t, d_t\}_{t=1}^{\infty}$. If the sequence $\{c_t, d_t\}_{t=1}^{\infty}$ is not converging to the steady state, we repeat the search process with another guess of c_1.[13]

Figure 1 shows that when the tax policy is applied, there would be an increase in both the capital-to-output ratio and durable-to-nondurable ratio, which results in welfare improvement as shown in Proposition 1. Over time, the equilibrium converges

[13] The numerical analyses in this subsection were performed with MATLAB 9. All MATLAB codes can be downloaded from minwook.host22.com/code/DurableTax.

to the steady state ($k^* = 3$ and $d_t^*/c_t^* = 1.31$). We measure the welfare gain from the tax policy in terms of period-1's nondurable consumption goods. The consumer is indifferent between the equilibrium consumption without the tax policy plus a one-time additional nondurable consumption subsidy and that with the tax policy. Specifically, the welfare measure (g) satisfies the following equation:

$$\begin{aligned} & u(c_1^* + c_1^* g) + v(d_1^*) + \sum_{\tau=1}^{\infty} \delta^\tau \left(u(c_{t+\tau}^*) + v(d_{t+\tau}^*) \right) \\ = \; & u(c_t^+) + \sum_{\tau=1}^{\infty} \delta^\tau u \left(u(c_{t+\tau}^+) + v(d_{t+\tau}^+) \right), \end{aligned} \tag{42}$$

where $(c_1^*, d_1^*, c_2^*, d_2^*, ...)$ is the equilibrium consumption without the tax policy and $(c_1^+, d_1^+, c_2^+, d_2^+, ...)$ is the equilibrium with the tax policy. The numerical analysis shows that the welfare gain (g) is 6.9%. It also shows that in the steady state, the consumption tax rates for nondurable (τ_c^*) and durable (τ_d^*) goods are 42.8% and 5.8%, respectively. The large gap between the two tax rates imply that uniform consumption taxation is not an efficient way to curb consumer present bias.

5 Conclusion

This paper incorporates the purchase of durable goods into Laibson's hyperbolic discounting model and calculates the optimal consumption rates for nondurable and durable goods, respectively. The theoretical and numerical results indicate that the optimal consumption tax rates for the two types of goods are considerably different, which implies that a uniform consumption tax cannot achieve first-best allocations.

Even though an important topic in macroeconomics has been durable goods, applications of durable goods in the literature of a macroeconomics hyperbolic-discounting model has been scarce . To the best of my knowledge, this paper is the first attempt to investigate optimal taxation with durable goods. More future research should focus on this important topic considering that consumers' consumption behavior is significantly different for durable and nondurable goods purchases, especially under hyperbolic discounting.

Appendices

A Proof of Lemma 1

This proof is equivalent to proving that

$$A_t = U_1(c_{t+1}, d_{t+1}).$$

By Eqs. (20-21), we know that $A_{t+1} = U_1(c_{t+2}, d_{t+2})$. As in Eq. (19), we have

$$\begin{aligned} A_t &= \lambda_{t+1}\left\{e_{t+1}U_1(c_{t+1}, d_{t+1}) + (1-e_{t+1})\left(U_2(c_t, d_t) + \delta(1-\eta)A_{t+1}\right)\right\} \\ &\quad + (1-\lambda_{t+1})\,\delta R\left(\overline{k}_{t+2}\right)A_{t+1}. \end{aligned} \tag{43}$$

Eqs. (20-21) imply that $A_{t+1} = U_1(c_{t+2}, d_{t+2})$. Plugging $A_{t+1} = U_1(c_{t+2}, d_{t+2})$ into Eq. (43), we have

$$\begin{aligned} A_t &= \lambda_{t+1}\left\{\begin{array}{c} e_{t+1}U_1(c_{t+1}, d_{t+1}) + \\ (1-e_{t+1})\left(U_2(c_{t+1}, d_{t+1}) + \delta(1-\eta)U_1(c_{t+2}, d_{t+2})\right) \end{array}\right\} \\ &\quad + (1-\lambda_{t+1})\,\delta R\left(\overline{k}_{t+2}\right)U_1(c_{t+2}, d_{t+2}). \end{aligned} \tag{44}$$

Because we know $\delta R(\overline{k}_{t+2})U_1(c_{t+2}, d_{t+2}) = U_2(c_t, d_t) + \delta(1-\eta)U_1(c_{t+2}, d_{t+2})$ from Eqs. (20-21), from Eq. (44) we can show that $A_t = U_1(c_{t+1}, d_{t+1})$ as follows

$$\begin{aligned} A_t &= \lambda_{t+1}\left\{e_{t+1}U_1(c_{t+1}, d_{t+1}) + (1-e_{t+1})\,\delta R(\overline{k}_{t+2})U_1(c_{t+2}, d_{t+2})\right\} \\ &\quad + (1-\lambda_{t+1})\,\delta R\left(\overline{k}_{t+2}\right)U_1(c_{t+2}, d_{t+2}) \\ &= U_1(c_{t+1}, d_{t+1}). \end{aligned}$$

Plugging $A_t = U_1(c_{t+1}, d_{t+1})$ into Eqs. (16-17), we have

$$u_1(c_t, d_t) = (1+\tau_{n,t})\,\beta\delta R\left(\overline{k}_{t+1}\right)U_1(c_{t+1}, d_{t+1})$$

and

$$u_2(c_t, d_t) + \beta\delta R(\overline{k}_{t+1})(1-\eta) = (1+\tau_{d,t})\,\beta\delta R(\overline{k}_{t+1})U_1(c_{t+1}, d_{t+1}).$$

B Proof of Proposition 1

We can prove it in a recursive way. In the last period T, an efficient allocation should satisfy the following equation:

$$\frac{U_1(c_T, d_T)}{U_2(c_T, d_T) + \delta\left(1-\varphi\right)\left(1-\eta\right)U_1(c_T, d_T)} = 1.$$

Therefore, the tax rates for durable and nondurable good should be the same, i.e., $\tau_{n,T} = \tau_{d,T}$.

In period $T-1$, the Euler equations are

$$U_1(c_{T-1}, d_{T-1}) = \left(1+\tau_{n,T-1}\right)\beta\delta R\left(\bar{k}_T\right)U_1(c_T, d_T) \tag{45}$$

and

$$U_2(c_{T-1}, d_{T-1}) + \beta\delta(1-\eta)U_1(c_T, d_T) = \left(1+\tau_{d,T-1}\right)\beta\delta R\left(\bar{k}_T\right)U_1(c_T, d_T) \tag{46}$$

By choosing the tax rate in period T-1 as

$$\tau_{n,T-1} = \frac{1}{\beta} - 1 \tag{47}$$

and

$$\tau_{d,T-1} = \frac{1}{\beta}\frac{R\left(\bar{k}_T\right) - (1-\beta)(1-\eta)}{R\left(\bar{k}_T\right)} - 1, \tag{48}$$

the Euler equations in Eqs. (45-46) become

$$U_1(c_{T-1}, d_{T-1}) = \delta R\left(\bar{k}_T\right)U_1(c_T, d_T) \tag{49}$$

and

$$U_2(c_{T-1}, d_{T-1}) = \delta\left\{R\left(\bar{k}_T\right) - (1-\eta)\right\}U_1(c_T, d_T), \tag{50}$$

which are the same as the Euler equations under efficient allocations in Eqs. (12-13).

By Lemma 1, given the Euler equations (49-50), period T-2's Euler equations are

$$u_1(c_{T-2}, d_{T-2}) = \left(1+\tau_{n,T-2}\right)\beta\delta R\left(\bar{k}_{T-1}\right)u_1(c_{T-1}, d_{T-1})$$

and

$$u_1(c_{T-2}, d_{T-2}) + \beta\delta R(\bar{k}_{T-1})(1-\eta) = \left(1+\tau_{n,T-2}\right)\beta\delta R\left(\bar{k}_{T-1}\right)u_1(c_{T-1}, d_{T-1})$$

By choosing the tax rates in period T-2 as

$$\tau_{n,T-2} = \frac{1}{\beta} - 1 \tag{51}$$

and

$$\tau_{d,T-2} = \frac{R\left(\overline{k}_{T-1}\right) - (1-\beta)(1-\eta)}{\beta R\left(\overline{k}_{T-1}\right)} - 1, \tag{52}$$

we have the Euler equations:

$$U_1(c_{T-2}, d_{T-2}) = \delta R\left(\overline{k}_T\right) U_1(c_{T-1}, d_{T-1}) \tag{53}$$

and

$$U_2(c_{T-2}, d_{T-2}) = \delta \left\{ R\left(\overline{k}_{T-1}\right) - (1-\eta) \right\} U_1(c_{T-1}, d_{T-1}), \tag{54}$$

which are also the same as the Euler equations under efficient allocations in Eqs. (12-13). Repeating this process, we obtain the optimal taxation:

$$1 + \tau_{n,t} = \frac{1}{\beta}, \tag{55}$$

and

$$1 + \tau_{d,t} = \frac{R\left(\overline{k}_{t+1}\right) - (1-\beta)(1-\eta)}{\beta R\left(\overline{k}_{t+1}\right)}. \tag{56}$$

for $t = 1, ..., T-1$.

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