

# Addiction, Present-Bias, and Self-Restraint

Lawrence Jin  
National University of Singapore

Minwook Kang  
Korea University

June 19, 2022

## Abstract

We study the economic rationale for people to engage in self-restraint. Specifically, we show that when a good is addictive and harmful, forward-looking present-biased consumers have an incentive to restrict their current consumption – below the level at which marginal utility of consumption equals marginal cost – in order to curb their future consumption of the harmful good. If the incentive is sufficiently strong, present bias may encourage consumers to consume less of the addictive bad today. Finally, we show that a self-restraint incentive can coexist with a sin tax on the addictive bad.

**Keywords:** Addiction; Present-Bias; Hyperbolic discounting; Self-Restraint

**JEL codes:** D91; D11

---

We appreciate helpful comments and constructive remarks from two anonymous referees and the editor (Charles J. Courtemanche). This research is supported by a Korea University Grant (K2123771) and Ministry of Education, Singapore (Start-up Grant). All errors are our own.

*“Complete abstinence is easier than perfect moderation” – St. Augustine*

## 1. Introduction

In economics, the act of restricting one’s own consumption has largely been understood through the lens of self-control and time consistency. For example, an agent holds back from indulging today because she gives sufficient weight to the negative utility it generates in the future against the utility gain today. Self-control allows an agent to maximize utility in a time-consistent manner.

However, some people impose even stronger restrictions on their consumption. For example, many people go so far as to engage in no-sugar or no-carb diets, even though consuming them in moderate amounts is not harmful. Similarly, one of the authors of this paper has refused to start watching a popular and critically acclaimed Netflix series because of his fear of becoming addicted (binge watching). Thus, he only watches mediocre shows, even though he knows he would enjoy the former much more.

In this paper, we use an economic framework to model when and why people exercise self-restraint. We define self-restraint as an act of consuming *less* than the level at which the marginal utility of consumption equals the marginal cost. Specifically, we show that when a pleasurable good is addictive and generates harm in the future, forward-looking present-biased consumers have an incentive to consume less of the good today to curb their future selves from overconsuming the good.<sup>1</sup>

Consider a simple numerical example. A student has an exam in period 3 and has an opportunity to watch a newly released TV show in the first two periods. There is no cost to watching the show in period 1, but watching the show in period 2 will severely impact the student’s performance on the exam and will incur a cost of 5 utils in period 3. In addition, the show is addictive. Watching the show for the first time generates an immediate reward of 1 util, and his enjoyment increases to 3 utils if he watched the show in the previous period as well.

Let us assume linear utility. A time-consistent student (with  $\delta = 1$ ) will watch the show in period 1 and not watch the show in period 2. He does not

---

<sup>1</sup>Self-restraint is an example of a self-control method. Another common self-control method involves the use of a commitment device. In Section 3, we show that the incentive to engage in self-restraint can coexist with a commitment device such as a sin tax imposed on harmful goods.

watch it in period 2 because the reward is always less than the cost ( $3 < 5$  and  $1 < 5$ ), and he watches it in period 1 because the reward is higher than the cost ( $1 > 0$ ).

By contrast, a present-biased but sophisticated student may choose not to watch the show at all. Consider a quasi-hyperbolic consumer with  $\beta = 0.55$ . He knows that if he watches the show in period 1, he will end up watching the show in period 2, because the immediate reward is larger than the discounted future cost ( $3 > 5\beta = 2.75$ ). If he does not watch it in period 1, he will not watch it in period 2 ( $1 < 5\beta$ ). Therefore, the only two viable options are to either watch the show in both periods or not watch it in both periods. Watching TV in both periods leads to a discounted period-1 utility of  $1 + 3\beta - 5\beta = -0.1 < 0$ , so he chooses to refrain from watching the show entirely in both periods.<sup>2</sup>

The results may appear somewhat counterintuitive in that sophisticated present-biased agents have an incentive to reduce the consumption of the addictive bad today to regulate their future consumption. This incentive appears to be particularly strong if the bad is highly addictive, or if future consumption is significantly harmful. While this scenario is an extreme (corner-solution) example to illustrate a self-restraint motive, in Section 2 we demonstrate the existence of a self-restraint incentive in a more general case with interior solutions.

Interestingly, we also find that the incentive to exercise self-restraint exists even in the presence of commitment devices. Commitment devices, such as a sin tax for harmful goods, are external tools that allow people to regulate their future consumption. Similarly, self-restraint helps regulate future consumption. However, there are two subtle but important distinctions between commitment devices and self-restraint. First, self-restraint is only a viable option when the good is addictive because it works by reducing the future stock of the addictive good. In contrast, commitment devices typically work by increasing the price of future consumption. Second, self-restraint is an internal method of regulating future consumption that does not rely on the availability of external devices or interventions. As a result, sophisticated present-biased consumers have an incentive to engage in self-restraint with addictive bads even when external commitment devices (e.g., a sin tax) are present. In Section 3, we show that a self-restraint incentive can coexist with a sin tax in the context of the addictive bad.

Our findings generate interesting implications. First, our results demonstrate

---

<sup>2</sup>Note that a naive, present-biased student would end up watching TV in both periods.

that present-biased consumers have an internal incentive to exercise self-restraint. For many years, governments and organizations have provided external commitment devices (e.g., a sin tax) to help mitigate the problem of over-consumption of harmful goods. Our findings suggest that increasing the sophistication of present-biased consumers may be an effective and complementary method to mitigate over-consumption by encouraging present-biased consumers to exercise self-restraint.

Another interesting implication is what happens when cessation products, such as nicotine replacement products, or electronic cigarettes for quitting smoking, are introduced in the market. Cessation products make it easier to quit an addictive good, and thus reduce the self-restraint incentive that sophisticated present-biased consumers have towards the addictive good. Ultimately, however, the new cessation products could potentially increase the consumption of the addictive good for some sophisticated present-biased consumers. That is, cessation products could help current smokers quit, but it could also increase smoking initiation among non-smokers who previously exercised self-restraint.

Much of the economics literature on present bias has focused on why people over-consume harmful addictive goods (or under-consume beneficial goods). Our paper provides novel insights on why people sometimes aggressively under-consume (over-consume) such goods.

Our paper builds on the rational addiction literature pioneered by Becker and Murphy (1988), and Gruber and Köszegi (2001; 2004), in particular, who incorporate hyperbolic discounting into the addiction model. Gruber and Köszegi (2001; 2004) focus on the myopic mistake of present-biased agents, namely, their tendency to overconsume addictive bads (compared to the paternalistic preference of exponential discounting) due to a self-control problem. Our paper focuses on a different phenomenon that arises from the self-control problem: present-biased consumers anticipate that their future selves will overconsume the addictive bad (compared to the current selves' present-biased preference), and as a result, they are willing to consume less today in order to regulate future consumption. Our results do not rely on normative assumptions about the optimal intertemporal preference.

This paper also contributes to the literature on a hyperbolic-discounting model (Strotz 1956; Phelps and Pollack 1968; Pollack 1968; Laibson 1997). The literature has identified various commitment devices such as illiquid financial assets (Laibson 1997; Kocherlakota 2001), a minimum savings rule (Amador et

al. 2006), commitment savings contract (Bond and Sigurdsson 2018), sin tax (O’Donoghue and Rabin 2006) and information restriction (Bénabou and Tirole 2002; 2004). A growing number of empirical studies have also investigated the demand for commitment devices for addictive goods such as smoking and alcohol (Gruber and Mullainathan, 2005; Bernheim et al., 2016; Hinnosaar, 2016; Schilbach, 2019).<sup>3</sup> In this paper, we demonstrate that self-restraint is a viable commitment method that does not rely on the availability of external commitment devices.

Finally, this paper contributes to the understanding of how our decisions are affected by present bias. A growing body of empirical literature has documented evidence of present bias across many decisions we make, including smoking (Gruber and Köszegi 2001; 2004), exercising (Dellavigna and Malmendier 2006), eating (Ruhm 2012), energy consumption (Schleich et al. 2019; Werthschulte and Loschel 2021), demand deposits (Kang 2020), aggregate savings (Kang 2021), and participation in welfare programs (Fang and Silverman, 2009). Bradford et al. (2017) show that individuals’ time preferences are significant predictors of their behaviors across multiple domains, including health, energy use, and financial decisions.

The remainder of this paper is organized as follows. Section 2 develops a simple three-period addictive/non-addictive goods model with hyperbolic discounting. Section 3 derives an inverse demand function for an addictive good and shows that consumers, in general, have a self-restraint incentive in consuming the addictive bad. Section 4 compares the self-restraint incentive with self-commitment tools that have been widely researched in economics. Section 5 concludes.

## 2. The model

### 2.1. The period-utility

Following Becker and Murphy (1988) and Gruber and Köszegi (2001), we assume that the period-utility function at period  $t$  is

$$U_t = U(a_t, c_t, S_t) = v(a_t, S_t) + u(c_t) \quad (1)$$

---

<sup>3</sup>For an excellent survey paper on commitment devices under present bias, see Bryan et al. (2010).

where  $v_1 > 0, v_2 < 0, v_{12} > 0, u' > 0$ . To guarantee interior solutions, we assume that  $U_t$  is concave and  $\lim_{a_t \rightarrow 0} v_1(a_t, S_t) = \infty, \lim_{a_t \rightarrow \infty} v_1(a_t, S_t) = 0, \lim_{c_t \rightarrow 0} u'(c_t) = \infty, \lim_{c_t \rightarrow \infty} u'(c_t) = 0$ .  $S_t$  represents the stock of an addictive good at period  $t$ . We have:

$$S_{t+1} = (S_t + a_t)(1 - d), \quad (2)$$

where  $d$  represents the depreciation rate. Period-1 stock,  $S_1$ , is exogenously given in this model.

The key assumptions of the addiction model are that the stock of the addictive good increases the marginal utility of future consumption (i.e., reinforcement, or  $v_{12} > 0$ ) and decreases future period utility (i.e., adverse health effect, or  $v_2 < 0$ ).

## 2.2. Hyperbolic discounting and intertemporal utility

We assume that an individual lives three periods. In each period, the individual maximizes the intertemporal utility defined by the hyperbolic discounting framework. Under the assumption that the individual is sophisticated, we solve the maximization problem using backward induction.

We assume that in each period, there is a fixed income. Becker and Murphy (1988) hold the marginal utility of wealth constant when analyzing price changes, which in practice is similar to assuming no savings.  $y_1, y_2$  and  $y_3$  represent incomes in periods 1, 2, and 3, respectively. In each period, the price of the non-addictive good is one, and the price of the addictive good is  $p_t$  in periods  $t = 1, 2, 3$ . We follow Gruber and Köszegi (2001; 2004) and assume no credit market in our model, and that some exogenously given income is consumed in each period. This assumption makes the model more tractable, and relaxing this assumption will not impact the main results.<sup>4</sup>

In period 3, given the period-3 stock of the addictive good,  $S_3$ , the consumer

---

<sup>4</sup>With the credit market, the consumer will move resources across time periods. Thus, the expenditure in each period would not necessarily be the same as the period's income, i.e.,  $c_t + p_t a_t \neq y_t$ . However, we have the expenditure as  $c_t + p_t a_t = y'_t$ , where  $(y'_1, y'_2, y'_3)$  are equilibrium expenditures in an economy with the credit market, and the present value of the total expenditure is equal to the present value of the incomes in the three periods such that

$$y_1 + \frac{y_2}{1 + r_2} + \frac{y_3}{(1 + r_2)(1 + r_3)} = y'_1 + \frac{y'_2}{1 + r_2} + \frac{y'_3}{(1 + r_2)(1 + r_3)},$$

where  $r_2$  and  $r_3$  are real interest rates in periods 2 and 3, respectively. In this case, the equilibrium allocation replacing  $(y_1, y_2, y_3)$  with  $(y'_1, y'_2, y'_3)$  is equivalent to those with  $(y_1, y_2, y_3)$  and the credit market. Therefore, we can derive the demand function from the Euler equations in terms of the addictive good with incomes  $(y'_1, y'_2, y'_3)$ , which lead to the same results as in Proposition 1.

solves the following maximization problem:

$$\max_{a_3, c_3} v(a_3, S_3) + u(c_3) \quad (3)$$

subject to

$$p_3 a_3 + c_3 = y_3.$$

From the maximization problem of *Eq.* (3), we have  $a_3$  and  $c_3$  as a function of  $S_3$  (see Appendix A for the detailed mathematical derivation). Denote  $\bar{a}_3(S_3)$  and  $\bar{c}_3(S_3)$  as the solution to the maximization problem of *Eq.* (3).

We use the  $\beta$ - $\delta$  hyperbolic discounting model in this paper. In period 2, given the period-2 stock of the addictive good,  $S_2$ , the consumer solves the following maximization problem:

$$\max_{a_2, c_2} v(a_2, S_2) + u(c_2) + \beta\delta \{v(a_3, S_3) + u(c_3)\} \quad (4)$$

subject to

$$p_2 a_2 + c_2 = y_2.$$

By replacing  $a_3$  and  $c_3$  with  $\bar{a}_3(S_3)$  and  $\bar{c}_3(S_3)$ , we can solve the maximization problem of *Eq.* (4). Denote  $\bar{a}_2(S_2)$  and  $\bar{c}_2(S_2)$  as the solution to the maximization problem of *Eq.* (4).

Finally, in period 1, given the period-1 stock of the addictive good,  $S_1$ , the consumer solves the following maximization problem:

$$\max_{a_1, c_1} v(a_1, S_1) + u(c_1) + \beta\delta \{v(a_2, S_2) + u(c_2)\} + \beta\delta^2 \{v(a_3, S_3) + u(c_3)\} \quad (5)$$

subject to

$$p_1 a_1 + c_1 = y_1.$$

We can solve for  $a_1$  and  $c_1$  by plugging  $(\bar{a}_2(S_2), \bar{c}_2(S_2))$  and  $(\bar{a}_3(S_3), \bar{c}_3(S_3))$  into the period-1 maximization problem in *Eq.* (5).

### 3. Main result

In this section, we present the main result of this paper. If the addiction exhibits complementarity (i.e., the past accumulation of the addictive stock increases the current consumption of the addictive good), the consumer has an incentive to engage in self-restraint. We show that this complementarity prop-

erty is derived from the key assumption of reinforcement ( $v_{12} > 0$ ) in our model.

To show the existence of a positive self-restraint incentive, we derive the consumer's inverse demand function of the addictive good in period 1 ( $a_1$ ). Through the inverse demand function, we can identify the relationship between the marginal cost ( $p_1$ ; i.e., how much the consumer is willing to pay to buy one unit of the addictive good) and the marginal benefit (i.e., the gain in lifetime utility by consuming one unit of the addictive good). The self-restraint incentive makes the marginal utility higher than the marginal cost in the inverse demand function, and as a result, the consumer decreases consumption of the addictive good.

To derive the inverse demand function of  $a_1$ , we solve the maximization problems using backward induction. From the period-3 and period-2 maximization problems in *Eqs.* (3) and (4), we can derive  $(\bar{a}_2(S_2), \bar{c}_2(S_2))$  and  $(\bar{a}_3(S_3), \bar{c}_3(S_3))$ , respectively. Plugging them into the period-1 maximization problem in *Eq.* (3), we have the following Euler equation: (for the detailed mathematical derivation, see the proof of Proposition 1 in Appendix A)

$$\begin{aligned}
& v_1(a_1, S_1) + \beta\delta \left\{ \underbrace{v_2(a_2, S_2)(1-d) + (v_1(a_2, S_2) - p_2 u'(c_2)) \bar{a}'_2(S_2)(1-d)}_{\text{Term I}} \right\} \\
& + \beta\delta^2 \left\{ \underbrace{v_1(a_3, S_3) \bar{a}'_3(S_3)(1-d)^2 + u'(c_3) \bar{c}'_3(S_3)(1-d)^2}_{\text{Term II}} \right. \\
& \quad \left. + v_2(a_3, S_3)(1-d)^2 + v_2(a_3, S_3) \bar{a}'_2(S_2)(1-d)^2 \right\} \\
& = p_1 \times u'(c_1)
\end{aligned} \tag{6}$$

In the Euler equation of *Eq.* (6), the two terms I and II exist by the complementarity property of addiction, i.e.,  $\bar{a}'_2(S_2) > 0$ . Thus, from terms I and II, we derive the self-restraint incentive in the inverse demand function. From *Eq.* (6) and the Euler equations of periods 2 and 3, we have the following inverse demand function: (see Appendix A).

$$\begin{aligned}
p_1 & = \underbrace{\frac{v_1(a_1, S_1) + \beta\delta v_2(a_2, S_2)(1-d) + \beta\delta^2 v_2(a_3, S_3)(1-d)^2}{u'(c_1)}}_{\text{(Relative) Marginal utility}} \\
& \quad - \underbrace{\left\{ -\beta\delta^2 (1-\beta) v_2(a_3, S_3) \bar{a}'_2(S_2) / u'(c_1) \right\}}_{\text{Self-restraint incentive}}
\end{aligned} \tag{7}$$

Because we can express  $a_2$ ,  $a_3$ ,  $S_2$ ,  $S_3$ , and  $c_1$  as a function of  $a_1$  through the response functions  $(\bar{a}_2(S_2), \bar{c}_3(S_3), \bar{a}_3(S_3)$  and  $\bar{c}_3(S_3))$ , *Eq.* (7) basically repre-



sents the relationship between  $a_1$  and  $p_1$ , which is the inverse demand function of the period-1 consumption of the addictive good. The inverse demand function in Eq. (7) can be divided into two parts. The first part is the marginal lifetime utility and the second part is the self-restraint incentive.

There are three terms in the marginal utility component of Eq. (7). Consuming one more unit of the addictive good increases the current utility by  $v_1(a_1, S_1)$ . It also increases the future addictive stock with depreciation. Consuming one unit of the addictive good in the first period increases the addictive stock in periods 2 and 3 by  $(1 - d)$  and  $(1 - d)^2$ , respectively. These increases in the addictive stock decrease the utility in future periods since we have  $v_2(a_2, S_2) < 0$  and  $v_2(a_3, S_3) < 0$ .

The second component of Eq. (7), the self-restraint incentive, is derived from two important characteristics: time inconsistency ( $\beta$ ) and the impact of the addictive stock on future period utility ( $v_2(a_3, S_3)$ ). If either are zero, the consumer would have no incentive to engage in self-restraint. If the self-restraint incentive is strictly positive in Eq. (7), the demand price of the addictive good ( $p_1$ ) is lower than its marginal utility. In this case, the consumer consumes the addictive good less than the amount where the marginal cost of consumption ( $p_1$ ) equals the marginal utility.

Eq. (7) indicates that if  $\bar{a}'_2(S_2) > 0$  (i.e., there is adjacent complementarity), then a higher level of addictive stock increases consumption of the addictive good. This complementarity can be considered a natural result of addiction. Therefore, we have the following Proposition:

**Proposition 1** *If an addictive good is complementary to the addictive stock (or  $\bar{a}'_2(S_2) > 0$ ) and the consumer is time-inconsistent (or  $\beta < 1$ ), the consumer has a positive self-restraint incentive to decrease consumption of the addictive good. That is, the marginal lifetime utility of the addictive good in period 1 is larger than the demand price of the addictive good.<sup>5</sup>*

**Proof:** See Appendix A.

To further analyze the self-restraint incentive, we consider an example of quadratic utility (Gruber and Köszegi, 2001; 2004) in which a closed form solu-

---

<sup>5</sup>We can model partial/full naivete following O'Donohue and Rabin (1999). Specifically, assuming that the period-1 self believes period-2 self's hyperbolic discount factor is  $b \in [\beta, 1]$ , the consumer with  $b = \beta$ ,  $b \in (\beta, 1)$  and  $b = 1$  can be interpreted as fully sophisticated, partially naive, and fully naive, respectively. Solving the period-2 maximization problem by

tion exists. With a utility function  $U(a_t, S_t, c_t) = 2a_t - \frac{a_t^2}{2} + \frac{2}{5}a_t S_t - S_t + c_t$ ,<sup>6</sup> incomes  $(y_1, y_2, y_3) = (1, 1, 1)$ , zero depreciation ( $d = 0$ ), and discounting factors  $\beta = 1/2$  and  $\delta = 1$ , we have the following inverse demand function from the Euler equation of Eq. (7). For the detailed mathematical derivation, see Appendix C.<sup>7</sup>

$$p_1 \times \underbrace{1}_{u'(c_1)=1} = \underbrace{(1.413 - 0.774a_1)}_{\text{Marginal Utility} = MU(a_1)} - \underbrace{(0.0624 - 0.0318a_1)}_{\text{Self-Restraint Incentive} = SR(a_1)} \quad (11)$$

In Figure 1, the equilibrium first-period consumption of the addictive good is  $a_1 = 0.47$  when  $p_1 = 1$ . However, without any self-restraint incentive, the equilibrium consumption would be higher, at  $a_1 = 0.53$ . The intuition is that the agent consumes *less* due to the self-restraint incentive to regulate future consumption of the addictive good. In this example, there is adjacent complementary, since  $\bar{a}_2(S_2) = 0.761 + 0.522 \times S_2$ . Hence, based on Proposition 1, there is a positive self-restraint incentive.

Proposition 1 shows the necessary and sufficient condition for the existence of a self-restraint incentive, which is that the sign of  $\bar{a}'_2(S_2)$  is positive. The key implication of the addiction is that when the past addictive-good consumption

---

replacing  $\beta$  with  $b$  in the proof of Proposition 1, we get the following Euler equation:

$$p_1 \times u'(c_1) = \underbrace{\frac{v_1(a_1, S_1) + \beta\delta v_2(a_2, S_2)(1-d)}{+\beta\delta^2 v_2(a_3, S_3)(1-d)^2}}_{\text{Marginal utility}} - \underbrace{\left\{ - (1-b)\beta\delta^2 v_2(a_3, S_3)\bar{a}'_2(S_2)/u'(c_1) \right\}}_{\text{Self-restraint incentive}} \quad (8)$$

Eq. (8) indicates that as the degree of naivety increases (i.e.,  $b$  increases), the self-restraint incentive decreases. It becomes zero if the consumer is fully naive (i.e.,  $b = 1$ ).

<sup>6</sup>Even though this example violates the assumption of strict concavity of the utility function, there exists an interior solution such that all the first- and second-order conditions are satisfied.

<sup>7</sup>With a general form of the quadratic utility function in Gruber and Koszegi (2004),

$$U(a_t, S_t, c_t) = \tau a_t - \frac{a_t^2}{2} + k a_t S_t - S_t + c_t,$$

the marginal utility ( $MU(a_1)$ ) and self-restraint incentive ( $SR(a_1)$ ) are expressed as the following closed-form solutions:

$$MU(a_1) = \frac{a_1(\beta^2 k^3 + \beta(k+3)k^2 - 1) - \beta^2 k(k(p_3 - \tau - 1) + 1) - \beta(k^2 p^2 + k(p_2 + p_3 - 2\tau) + 2) + \tau}{1 - \beta k^2} \quad (9)$$

$$SR(a_1) = \frac{(1 - \beta)\beta k(1 + \beta k)(1 - a_1 k^2(1 + k) - k^2(\tau - p_2) - k(\tau - p_3))}{(1 - \beta k^2)^2} \quad (10)$$

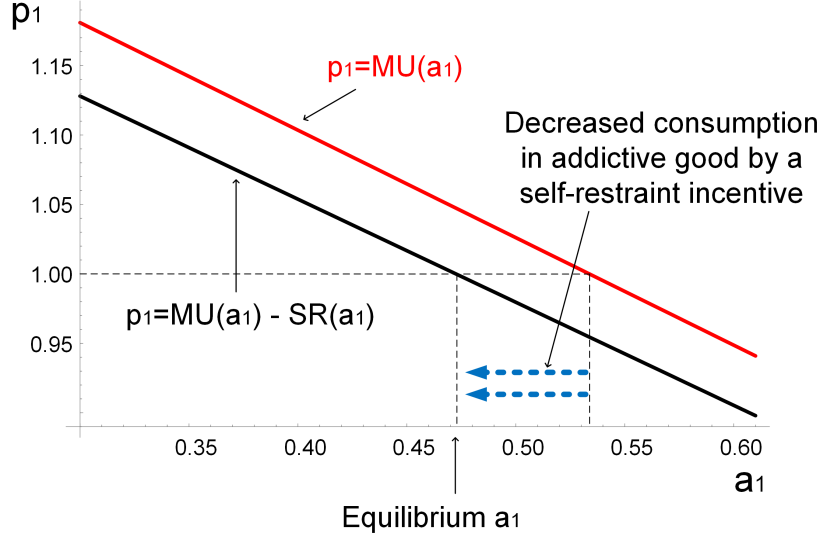


Figure 1: The inverse demand curve for the addictive good in period 1 ( $a_1$ ) in the leading example with quadratic utility function.

is high, the current consumption increases, which means  $\bar{a}'_2(S_2) > 0$  (adjacent complementarity). This implies that the future addictive consumption is complementary to today's consumption of the addictive good. This complementarity is directly derived from the property of addiction: that the consumption of an addictive good increases the marginal utility of future consumption. Specifically, we can derive  $\bar{a}'_2(S_2)$  as:<sup>8</sup>

$$\bar{a}'_2(S_2) = -\frac{v_{12}(a_2, S_2) + \beta\delta v_{12}(a_3, S_3)\bar{a}'_3(S_3)(1-d) + \beta\delta v_{22}(a_3, S_3)(1-d)}{v_{11}(a_2, S_2) + u''(c_2)p_2^2}. \quad (12)$$

From *Eq. (12)*, we can see that  $v_{12}(a_2, S_2) > 0$  is the main force that ensures  $\bar{a}'_2(S_2) > 0$  (i.e., addiction is complementary to the stock of the addictive good). The term  $\beta\delta v_{22}(a_3, S_3)(1-d)$  in *Eq. (12)* could be a negative impact on the complementary property. However, this impact is decreased by the discounting factor  $\beta\delta$  and depreciation  $(1-d)$ . Therefore, with a sufficiently small value of  $\beta\delta$  and a sufficiently large  $d$ , we can prove that the consumer behaves in a complementary way. As Gruber and Köszegi (2001) showed with a quadratic utility function example, in general, the direct impact of  $v_{12}(a_2, S_2)$  is stronger than the discounted (by  $\beta\delta(1-d)$ ) impact of  $\beta\delta v_{22}(a_3, S_3)$ .

From *Eq. (12)*, the sufficient condition for  $\bar{a}'_2(S_2) > 0$  is that  $v_{22}(a_3, S_3) = 0$ .

<sup>8</sup>For the detailed mathematical derivation, see the Proof of Corollary 1 in Appendix B.

Even though Becker and Murphy (1988) and Gruber and Köszegi (2001) assume that  $v_{22}(a_3, S_3) < 0$ , there is no obvious reason why the sign of  $v_{22}$  would be negative or positive. Becker and Murphy (1988) impose a negative sign of  $v_{22}$  to make the period utility strictly concave so an interior solution always exists. However, the key assumptions of addiction are reinforcement ( $v_{12} > 0$ ) and an adverse health effect ( $v_2 < 0$ ), but not whether the past consumption of the addictive good has an incremental or decremental negative impact ( $v_{22} < 0$  or  $v_{22} > 0$ ). Therefore, we have the following corollary:

**Corollary 1** *A sufficient condition for the addiction to be complementary is  $v_{12} > 0$  and  $v_{22} = 0$ .*

**Proof:** See Appendix B.

Corollary 1 suggests a sufficient condition for the addiction to be complementary, and thus for the existence of a positive self-restraint incentive. However, it does not necessarily mean that the consumer has a self-restraint incentive only if  $v_{22} = 0$ . For example, Gruber and Köszegi (2001) show that the complementary property can be demonstrated with a reasonable set of parameters with a quadratic utility function, even if  $v_{22} < 0$ .

This paper shows that addiction (i.e.,  $v_{12} > 0$ ) is a necessary condition for the consumer to have a positive self-restraint incentive. That is, this paper proves that a self-restraint incentive exists only if the good is addictive (that is  $v_{12} > 0$ ). However, if the good is not addictive (i.e.,  $v_{12} = 0$ ), the self-restraint incentive does not exist, as shown in Proposition 1 and Corollary 1. With the theoretical approach taken in this paper, there is no clear way to quantitatively measure the relation between the degree of addictiveness of the good and the magnitude of the self-constraint incentive. Nevertheless, the leading example shows how the magnitude of self-restraint is related to the degree of addictiveness. In the leading example, the degree of addictiveness is interpreted as the value of  $k$  where

$$U(a_t, S_t, c_t) = \tau a_t - \frac{a_t^2}{2} + k a_t S_t - S_t + c_t.$$

In this case, the self-constraint incentive is given by

$$SR(a_1) = -\frac{(1 - \beta)\beta k(1 + \beta k)(a_1 k^2(1 + k) + k^2(\tau - p_2) + k(\tau - p_3) - 1)}{(1 - \beta k^2)^2}. \quad (13)$$

From *Eq.* (13), we have

$$SR(a_1) = 0 \text{ where } k = 0, \quad (14)$$

and

$$\frac{\partial SR(a_1)}{\partial k} = (1 - \beta)\beta > 0 \text{ where } k = 0. \quad (15)$$

*Eqs.* (14-15) indicate that, at least around  $k = 0$ , self-constraint is increasing in the degree of addictiveness ( $k$ ), and there is no self-restraint incentive if there is no addictiveness (i.e.,  $k = 0$ ).

## 4. Commitment devices vs. Self-restraint incentives

The rationale for a present-biased consumer to engage in self-restraint is closely related to the demand for commitment devices in that both are intended to regulate future myopic decisions. However, they operate through a different channel: self-restraint curbs future consumption by reducing one's addictive stock in the future, while commitment devices curb future consumption by increasing the price. For example, in a model with unhealthy normal goods, O'Donoghue and Rabin (2006) propose revenue-neutral tax policies (also known as a sin tax) to improve the welfare of hyperbolic consumers.<sup>9</sup>

Would people have a self-restraint incentive when commitment devices are available? In this section, we use a sin tax as an example of a commitment device and show that present-biased consumers will still have an incentive to engage in self-restraint. This exercise also highlights the different mechanisms through which the two methods of regulating future myopic decisions work.

A sin tax or revenue-neutral tax policy is a commonly proposed commitment device in the literature. Specifically, the future period-2 self will need to pay  $(1 + T)p_2$ , where  $T$  is the proportional price tax, and will receive a lump sum transfer ( $X$ ) from an outside authority. The amount of lump-sum subsidy  $X$  is  $Tp_2a_2^*$ , where  $a_2^*$  is the equilibrium amount of  $a_2$ .

Although we use the term “tax policy,” the policy does not necessarily have to

---

<sup>9</sup>O'Donoghue and Rabin's (2006) main focus is to derive optimal taxation to maximize the social welfare function in a heterogeneous population. Therefore, the tax policy in O'Donoghue and Rabin (2006) is different from the self-selected commitment tool that is introduced in this section.

be implemented by the government. For example, a consumer can buy, a smoking cessation program or device from the private market. We simply show that as the tax ( $T$ ) increases from zero to a positive value, the consumption of the addictive bad ( $a_2$ ) decreases. If this policy improves the first-period intertemporal utility, the current self in period 1 is willing to buy the commitment device. We have the following first- and second-period budget constraints as:

$$c_1 + p_1 a_1 = y_1 - q, \quad (16)$$

$$c_2 + p_2(1 + T)a_2 = y_2 + X. \quad (17)$$

where  $q$  is the price of the commitment device.

We define that a commitment device exists if  $(T, q) \gg 0$  such that the period-1 intertemporal utility with  $(T, q) \gg 0$  is higher than that without (i.e.,  $(T, q) = 0$ ). This implies that the consumer is willing to pay  $q$  in period  $t$  to choose the tax policy  $T$ .

**Proposition 2** *Assuming that  $\beta < 1$ , the necessary and sufficient condition for the existence of a commitment device is  $v_2 < 0$  (adverse health cost) but it is irrelevant to the sign of  $v_{12}$  (reinforcement).*

**Proof:** See Appendix D.

Proposition 2 shows that even if  $v_{12} = 0$ , the consumer still has an incentive to use a commitment device to curb future myopic behavior. The result is similar to O'Donoghue and Rabin (2006) in which a revenue-neutral tax policy for an unhealthy good is proposed. Since they do not consider addiction, their model assumes  $v_{12} = 0$ .

The commitment device is distinct from self-restraint. Both are used to curb over-consumption in the future, but the commitment device increases the price of consumption, while self-restraint reduces the addictive stock of the future. It turns out that even in the presence of the commitment devices, present-biased consumers have an incentive to engage in self-restraint, as shown in the following proposition.

**Proposition 3** *Assume that  $\beta < 1$  and  $v_{22} = 0$ . The consumer has a self-restraint incentive regardless of the presence of any commitment device  $(T, q) \gg 0$ .*

**Proof:** See Appendix E.

Proposition 3 indicates that for any proposed commitment device above, a self-restraint incentive always exists. That is, the self-restraint incentive term in the inverse demand function of *Eq. (7)* is guaranteed to be strictly positive even with the availability of a self-commitment device. However, the result in Proposition 3 does not necessarily mean that the magnitude of a self-restraint incentive is unaffected by the use of a self-commitment device. The use of a self-commitment device is expected to affect the individual's incentive to exercise self-restraint.

As shown in Corollary 3,  $v_{22} = 0$  is a sufficient (but not necessary) condition for the existence of a positive self-restraint incentive. As discussed in the previous section, even if  $v_{22} \neq 0$ , a self-restraint incentive generally exists in this model. The same logic is applied regardless of the existence of a commitment device.

## 5. Conclusion

In the literature, present bias has been able to explain why people over-consume harmful addictive goods. In this paper, we demonstrate that present bias can also explain a seemingly opposite phenomenon. Specifically, present-biased and sophisticated consumers have an incentive to reduce their consumption of a harmful addictive good today to curb their future consumption. This incentive appears to be particularly strong when the good is highly addictive, or if future consumption is quite harmful. Using a quasi-hyperbolic discounting model framework, we show that this incentive can coexist with an external commitment device such as a sin tax on the addictive good.

Our findings have interesting empirical applications. First, our results demonstrate that present-biased consumers possess an incentive to exercise self-restraint, which is an internal mechanism of regulating their future self-control problem. For many years, governments and organizations have provided external commitment devices (e.g., a sin tax on harmful goods) to help mitigate over-consumption of harmful goods. Our results suggest that increasing sophistication among present-biased consumers may be another effective method to mitigate over-consumption by encouraging consumers to exercise self-restraint.

Second, our paper demonstrates why some sophisticated present-biased consumers may totally abstain from consuming an addictive good, even if consumption in moderation is not harmful or is even preferred at times. Examples include

people who adopt strict no-sugar or no-carb diets, or who practice total abstinence from pleasurable activities. These types of extreme abstinence behaviors may seem to be puzzling, but they can be explained by sophisticated consumers' desire to restrict their own consumption to extreme levels to regulate future over-consumption.

Finally, another interesting empirical prediction is what happens when cessation products, (e.g., nicotine replacement products for cigarette smoking) are introduced in the market. The cessation product makes it easier to quit the addictive good. However, although the introduction of a cessation product could reduce present-biased consumers' incentive to exercise self-restraint, they may subsequently increase consumption of the addictive good. This finding, if true, would have important implications for public policy. It would be interesting to test these empirical predictions and explore the policy implications in future studies.

## Appendices

### A. Proof of Proposition 1

Given  $S_3$ , the consumer's period-3 maximization problem is

$$\max_{a_3, c_3} v(a_3, S_3) + u(c_3),$$

subject to

$$p_3 a_3 + c_3 = y_3. \tag{18}$$

Because we assume that  $\lim_{a_t \rightarrow 0} v_1(a_t, S_t) = \infty$ ,  $\lim_{a_t \rightarrow \infty} v_1(a_t, S_t) = 0$ ,  $\lim_{c_t \rightarrow 0} u'(c_t) = \infty$ ,  $\lim_{c_t \rightarrow \infty} u'(c_t) = 0$ , there is an interior solution to Eq. (18) for any value of  $S_3 \in (0, \infty)$ .

From the first-order conditions of the period-3 maximization problem, we have

$$v_1(a_3, S_3) = u'(c_3)p_3. \tag{19}$$

Implicitly differentiating Eq. (19) with respect to  $S_3$ , we have

$$v_{11}(a_3, S_3)da_3 + v_{12}(a_3, S_3)dS_3 = u''(c_3)p_3dc_3 \tag{20}$$



From the budget constraint of *Eq. (18)*, we have

$$p_3 da_3 + dc_3 = 0. \quad (21)$$

From *Eqs (20)* and *(21)*, we have

$$v_{11}(a_3, S_3) da_3 + v_{12}(a_3, S_3) dS_3 = -u''(c_3) p_3^2 da_3, \quad (22)$$

which is, in turn, equivalent to

$$\frac{da_3}{dS_3} = \bar{a}'_3(S_3) = -\frac{v_{12}(a_3, S_3)}{v_{11}(a_3, S_3) + u''(c_3) p_3^2} > 0. \quad (23)$$

*Eq. (23)* implies that

$$\frac{dc_3}{dS_3} = \bar{c}'_3(S_3) < 0.$$

Given  $\bar{a}_3(S_3)$ ,  $\bar{c}_3(S_3)$  and  $S_2$ , the consumer's period-2 maximization problem is

$$\max_{a_2, c_2} v(a_2, S_2) + u(c_2) + \beta \delta \{v(a_3, S_3) + u(c_3)\}, \quad (24)$$

subject to

$$p_2 a_2 + c_2 = y_2. \quad (25)$$

From *(24)* and *(25)*, we have the following first-order conditions

$$\begin{aligned} & v_1(a_2, S_2) + \beta \delta \left\{ \begin{array}{l} v_1(a_3, S_3) \bar{a}'_3(S_3) + u'(c_3) \bar{c}'_3(S_3) \\ + v_2(a_3, S_3) (1 - d) \end{array} \right\} \\ & = p_2 u'(c_2) \end{aligned} \quad (26)$$

From *Eqs. (19)* and *(21)*, we have

$$v_1(a_3, S_3) \bar{a}'_3(S_3) + u'(c_3) \bar{c}'_3(S_3) = \{v_1(a_3, S_3) - u'(c_3) p_3\} \bar{a}'_3(S_3) = 0 \quad (27)$$

Thus, from *Eqs. (26)* and *(27)*, we have

$$v_1(a_2, S_2) + \beta \delta v_2(a_3, S_3) (1 - d) = u'(c_2) p_2. \quad (28)$$

From *Eqs. (25)* and *(28)*, we can derive  $\bar{a}_2(S_2)$  which satisfies the following

equation:

$$v_1(\bar{a}_2(S_2), S_2) + \beta\delta v_2(\bar{a}_3(S_3), S_3) (1 - d) = u'(y_2 - p_2\bar{a}_2(S_2))p_2, \quad (29)$$

where

$$S_3 = (1 - d) (S_2 + \bar{a}_2(S_2)).$$

Because we assume that  $\lim_{a_t \rightarrow 0} v_1(a_t, S_t) = \infty$ ,  $\lim_{a_t \rightarrow \infty} v_1(a_t, S_t) = 0$ ,  $\lim_{c_t \rightarrow 0} u'(c_t) = \infty$ ,  $\lim_{c_t \rightarrow \infty} u'(c_t) = 0$ , there is an interior solution to Eq. (29) for any value of  $S_2 \in (0, \infty)$ .

Given  $\bar{a}_2(S_2)$ ,  $\bar{c}_2(S_2)$ ,  $\bar{a}_3(S_3)$ ,  $\bar{c}_3(S_3)$  and  $S_1$ , the period-1 maximization problem is

$$\max_{a_1, c_1} v(a_1, S_1) + u(c_1) + \beta\delta \{v(a_2, S_2) + u(c_2)\} + \beta\delta^2 \{v(a_3, S_3) + u(c_3)\} \quad (30)$$

subject to

$$p_1 a_1 + c_1 = y_1.$$

From the first-order conditions from the period-1 maximization problem of (30), we have

$$\begin{aligned} & v_1(a_1, S_1) \quad (31) \\ & + \beta\delta \left\{ \underbrace{v_1(\bar{a}_2(S_2), S_2)\bar{a}'_2(S_2) (1 - d) + u'(\bar{c}_2(S_2))\bar{c}'_2(S_2) (1 - d)}_{\neq 0: a_1 \rightarrow S_2 \rightarrow a_2} \right. \\ & \quad \left. + \underbrace{v_2(\bar{a}_2(S_2), S_2) (1 - d)}_{a_1 \rightarrow S_2} \right\} \\ & + \beta\delta^2 \left\{ \underbrace{v_1(\bar{a}_3(S_2), S_3)\bar{a}'_3(S_3) (1 - d)^2 + u'(\bar{c}_3(S_3))\bar{c}'_3(S_3) (1 - d)^2}_{=0, a_1 \rightarrow S_3 \rightarrow a_3} \right. \\ & \quad \left. + \underbrace{v_2(\bar{a}_3(S_2), S_3) (1 - d)^2}_{a_1 \rightarrow S_3} \right. \\ & \quad \left. + \underbrace{v_2(\bar{a}_3(S_2), S_3)\bar{a}'_2(S_2) (1 - d)^2}_{a_1 \rightarrow S_2 \rightarrow a_2 \rightarrow S_3} \right\} \\ & = p_1 \times u'(c_1) \end{aligned}$$

From Eq. (27), we know that

$$v_1(\bar{a}_3(S_2), S_3)\bar{a}'_3(S_3) (1 - d)^2 + u'(\bar{c}_3(S_3))\bar{c}'_3(S_3) (1 - d)^2 = 0,$$

and, thus, *Eq. (31)* can be written as

$$\begin{aligned}
p_1 \times u'(c_1) &= v_1(a_1, S_1) + \beta\delta v_2(a_2, S_2)(1-d) + \beta\delta^2 v_2(a_3, S_3)(1-d)^2 \quad (32) \\
&+ \beta\delta \{v_1(a_2, S_2)\bar{a}'_2(S_2)(1-d) + u'(c_2)\bar{c}'_2(S_2)(1-d)\} \\
&+ \beta\delta^2 v_2(a_3, S_3)\bar{a}'_2(S_2)(1-d)^2
\end{aligned}$$

*Eq. (32)* can be divided into two part (Part A and Part B). Part A is the marginal utility which is

$$\text{Part A: } v_1(a_1, S_1) + \beta\delta v_2(a_2, S_2) + \beta\delta^2 v_2(a_3, S_3)(1-d). \quad (33)$$

and Part B is

$$\text{Part B: } \beta\delta \left\{ \begin{aligned} &v_1(a_2, S_2)\bar{a}'_2(S_2)(1-d) + u'(c_2)\bar{c}'_2(S_2)(1-d) \\ &+ \delta v_2(a_3, S_3)\bar{a}'_3(S_3)\bar{a}'_2(S_2)(1-d)^2. \end{aligned} \right\}. \quad (34)$$

From *Eq. (28)*, we have

$$v_1(a_2, S_2) + \beta\delta v_2(a_3, S_3)\bar{a}'_3(S_3)(1-d) = u'(c_2)p_2.$$

Using *Eq. (28)* and  $p_2\bar{a}'_2(S_2) + \bar{c}'_2(S_2) = 0$ , we can express Part B as

$$\begin{aligned}
&\beta\delta \{v_1(a_2, S_2)\bar{a}'_2(S_2)(1-d) + u'(c_2)\bar{c}'_2(S_2)(1-d) + \delta v_2(a_3, S_3)\bar{a}'_2(S_2)(1-d)^2\} \\
&= \beta\delta(1-d) \{v_1(a_2, S_2)\bar{a}'_2(S_2) + u'(c_2)\bar{c}'_2(S_2) + \delta v_2(a_3, S_3)\bar{a}'_2(S_2)(1-d)\} \\
&= \beta\delta(1-d) \{v_1(a_2, S_2)\bar{a}'_2(S_2) - p_2 u'(c_2)\bar{a}'_2(S_2) + \delta v_2(a_3, S_3)\bar{a}'_2(S_2)(1-d)\} \\
&= \beta\delta(1-d) \{-\beta\delta v_2(a_3, S_3)\bar{a}'_2(S_2)(1-d) + \delta v_2(a_3, S_3)\bar{a}'_2(S_2)(1-d)\} \\
&= \beta\delta^2(1-d)^2 v_2(a_3, S_3)(1-\beta)\bar{a}'_2(S_2). \quad (35)
\end{aligned}$$

From *Eqs. (32)* and *(35)*, we have

$$\begin{aligned}
p_1 \times u'(c_1) &= \underbrace{v_1(a_1, S_1) + \beta\delta v_2(a_2, S_2)(1-d) + \beta\delta^2 v_2(a_3, S_3)(1-d)}_{\text{Lifetime marginal utility}} \\
&+ \underbrace{\beta\delta^2(1-d)^2 v_2(a_3, S_3)(1-\beta)\bar{a}'_2(S_2)}_{\text{Self-restraint incentive}} \quad (36)
\end{aligned}$$

From *Eqs. (36)*, we know that the necessary and sufficient condition for the existence of a self-restraint incentives is that  $\bar{a}'_2(S_2) > 0$ .

## B. Proof of Corollary 1

Implicitly differentiating Eq. (28) with respect to  $S_2$ , we have

$$\begin{aligned} v_{11}(a_2, S_2)da_2 + v_{12}(a_2, S_2)dS_2 \\ + \beta\delta v_{12}(a_3, S_3)da_3 + \beta\delta v_{22}(a_3, S_3)dS_3 = u''(c_2)p_2dc_2. \end{aligned} \quad (37)$$

Implicitly differentiating the period-2 budget constraint, we have

$$p_2da_2 + dc_2 = 0. \quad (38)$$

From Eqs. (37) and (38), we have

$$\begin{aligned} v_{11}(a_2, S_2)da_2 + u''(c_2)p_2^2da_2 + v_{12}(a_2, S_2)dS_2 \\ + \beta\delta v_{12}(a_3, S_3)da_3 + \beta\delta v_{22}(a_3, S_3)dS_3 = 0. \end{aligned}$$

which is equivalent to

$$\bar{a}'_2(S_2) = -\frac{v_{12}(a_2, S_2) + \beta\delta v_{12}(a_3, S_3)\bar{a}'_3(S_3)(1-d) + \beta\delta v_{22}(a_3, S_3)(1-d)}{v_{11}(a_2, S_2) + u''(c_2)p_2^2}. \quad (39)$$

From Eq. (23), we know that  $\bar{a}'_3(S_3) > 0$ . Therefore,  $v_{22}(a_3, S_3) = 0$  is a sufficient condition for  $\bar{a}'_2(S_2)$  to be positive.

## C. An Example with Quadratic Utility

The period utility function is

$$U(a_t, S_t, c_t) = t \times a_t - \frac{a_t^2}{2} + k \times a_t \times S_t - S_t + c_t. \quad (40)$$

With the quadratic utility function in Eq. (40), we can derive  $\bar{a}_3(S_3)$  and  $\bar{c}_3(S_3)$ :

$$\bar{a}_3(S_3) = -p_3 + kS_3 + t, \quad \bar{c}_3(S_3) = y_3 - p_3 \times \bar{a}_3(S_3).$$

Plugging  $\bar{a}_3(S_3)$  and  $\bar{c}_3(S_3)$  into the period-2 maximization problem, we can get

$$\begin{aligned} \bar{a}_2(S_2) &= \frac{-kS_2 + p_2 - t + \beta(1 - k^2S_2 + kp_3 - kt)}{-1 + \beta k^2}, \\ \bar{c}_2(S_2) &= y_2 - p_2 \times \bar{a}_2(S_2). \end{aligned}$$

Plugging  $\bar{a}_2(S_2)$  and  $\bar{c}_2(S_2)$  into the period-1 maximization problem, we can derive the demand function of  $a_1$  in terms of  $p_1$ . With the parameter choices of  $t = 2, k = 0.4, \beta = 0.5, \delta = 1, d = 0, p_2 = p_3 = 1$ , we have the following demand function:

$$\bar{a}_1(p_1) = 1.81992 - 1.34743p_1. \quad (41)$$

From the demand function in Eq. (41), we have the following inverse demand function:

$$p_1 = 1.35066 - 0.742155a_1,$$

which can be divided into two parts from Eq. (36):

$$p_1 = \underbrace{(1.41304 - 0.773913a_1)}_{\text{Marginal utility} = \text{MU}(a_1)} - \underbrace{(0.0623819 - 0.031758a_1)}_{\text{Self-Restraint Incentive} = \text{SR}(a_1)}.$$

## D. Proof of Proposition 2

First, we prove that an increase in  $T$  from zero to a small positive value results in a decrease in  $a_2$ . The period-2 maximization problem is

$$\max_{a_2, c_2} v(a_2, S_2) + u(c_2) + \beta\delta \{v(a_3, S_3) + u(c_3)\} \quad (42)$$

subject to

$$c_2 + p_2(1 + T)a_2 = y_2 + X.$$

From the first-order conditions from the period-2 maximization problem of Eq. (42), we have

$$v_1(a_2, S_2) + \beta\delta v_2(a_3, S_3)(1 - d) = u'(c_2)p_2(1 + T). \quad (43)$$

The second-order conditions from the period-2 maximization problem of Eq. (42) are

$$v_{11}(a_2, S_2) + \beta\delta v_{12}(a_3, S_3)(1 - d)^2 \bar{a}'_3(S_3) + \beta\delta v_{22}(a_3, S_3)(1 - d)^2 < 0, \quad (44)$$

and

$$u''(c_2)p_2(1 + T) < 0. \quad (45)$$

Implicitly differentiating Eq. (43) with respect to  $T$ , we have

$$\begin{aligned} & v_{11}(a_2, S_2) \frac{da_2}{dT} + \beta \delta v_{12}(a_3, S_3) (1-d) \frac{da_3}{dT} + \beta \delta v_{22}(a_3, S_3) (1-d) \frac{dS_3}{dT} \\ = & u''(c_2) \frac{dc_2}{dT} p_2 (1+T) + u'(c_2) p_2. \end{aligned} \quad (46)$$

From period-2 budget constraint, we have

$$dc_2 + p_2(1+T)da_2 = p_2Tda_2,$$

which is equivalent to

$$dc_2 + p_2da_2 = 0. \quad (47)$$

Implicitly differentiating  $a_3 = \bar{a}_3(S_3)$  with  $T$ , we have

$$\frac{da_3}{dT} = \frac{da_2}{dT} (1-d) \bar{a}'_3(S_3). \quad (48)$$

Implicitly differentiating  $S_3 = (1-d)(S_2 + \bar{a}_2(S_3))$  with  $T$ , we have

$$\frac{dS_3}{dT} = \frac{da_2}{dT} (1-d). \quad (49)$$

From Eqs. (46-49), we have

$$\begin{aligned} & v_{11}(a_2, S_2) \frac{da_2}{dT} + \beta \delta v_{12}(a_3, S_3) (1-d)^2 \frac{da_2}{dT} \bar{a}'_3(S_3) + \beta \delta v_{22}(a_3, S_3) (1-d)^2 \frac{da_2}{dT} \\ = & -u''(c_2) \frac{da_2}{dT} p_2^2 (1+T) + u'(c_2) p_2, \end{aligned}$$

which is equivalent to

$$\frac{da_2}{dT} = \frac{u'(c_2) p_2}{\left\{ \begin{array}{l} v_{11}(a_2, S_2) + u''(c_2) \frac{da_2}{dT} p_2^2 (1+T) \\ + \beta \delta v_{12}(a_3, S_3) (1-d)^2 \bar{a}'_3(S_3) + \beta \delta v_{22}(a_3, S_3) (1-d)^2 \end{array} \right\}}. \quad (50)$$

From Eqs. (44-45), we know that  $\frac{da_2}{dT} > 0$  in Eq. 50.

The period-1 maximization problem is

$$\max_{a_1, c_1} v(a_1, S_1) + u(c_1) + \beta \delta \{v(a_2, S_2) + u(c_2)\} + \beta \delta^2 \{v(a_3, S_3) + u(c_3)\}. \quad (51)$$

Let  $U^{(1)}$  be the intertemporal utility in period 1, applying the envelop condition

in the maximization problem of Eq. (51), we have

$$\begin{aligned}\frac{dU^{(1)}}{dT} &= \beta\delta v_1(a_2, S_2)\frac{da_2}{dT} + \beta\delta u'(c_2)\frac{dc_2}{dT} \\ &\quad + \underbrace{\beta\delta^2 v_1(a_3, S_3)\frac{da_3}{dT} + \beta\delta^2 u'(c_3)\frac{dc_3}{dT}}_{=0} + \beta\delta^2 v_2(a_3, S_3)\frac{dS_3}{dT}. \\ &= \beta\delta v_1(a_2, S_2)\frac{da_2}{dT} + \beta\delta u'(c_2)\frac{dc_2}{dT} + \beta\delta^2 v_2(a_3, S_3)\frac{dS_3}{dT}.\end{aligned}\quad (52)$$

Because we have  $dc_3 + p_2 da_2 = 0$  from the budget constraint and  $\frac{dS_3}{dT} = (1-d)\frac{da_2}{dT}$ , Eq. (52) is

$$\begin{aligned}\frac{dU^{(1)}}{dT} &= \beta\delta v_1(a_2, S_2)\frac{da_2}{dT} - \beta\delta u'(c_2)p_2\frac{da_2}{dT} + \beta\delta^2 v_2(a_3, S_3)(1-d)\frac{da_2}{dT} \\ &= \beta\delta\frac{da_2}{dT}\{v_1(a_2, S_2) - u'(c_2)p_2 + \delta v_2(a_3, S_3)(1-d)\}.\end{aligned}\quad (53)$$

From the first-order conditions in the second period, we have

$$v_1(a_2, S_2) + \beta\delta v_2(a_3, S_3)(1-d) = u'(c_2)p_2(1+T).\quad (54)$$

From Eqs. (54) and (53), we have

$$\begin{aligned}\frac{dU^{(1)}}{dT} &= \beta\delta\frac{da_2}{dT}\{-\beta\delta v_2(a_3, S_3)(1-d) + u'(c_2)p_2T + \delta v_2(a_3, S_3)(1-d)\} \\ &= \beta\delta\frac{da_2}{dT}\{(1-\beta)\delta v_2(a_3, S_3)(1-d) + u'(c_2)p_2T\}.\end{aligned}\quad (55)$$

At  $T = 0$ , Eq. (55) becomes

$$\frac{dU^{(1)}}{dT}\Big|_{T=0} = \beta\delta^2(1-\beta)(1-d)v_2(a_3, S_3)\frac{da_2}{dT},\quad (56)$$

which is strictly positive if  $\beta < 1$  and  $v_2(a_3, S_3) < 0$ .

Now, we investigate the impact of the program cost  $q$  in the period-1 intertemporal utility. Applying the envelop theorem in the maximization problem of Eq. (51), we have

$$\frac{dU^{(1)}}{dq} = -\lambda < 0,\quad (57)$$

where  $\lambda \in (0, \infty)$  is the Lagrangian multiplier, which is the marginal utility of period-1 income. Because  $\lambda$  is a finite value, from Eqs. (53) and (57), there

exists  $(\Delta T, \Delta q) \gg 0$  where  $(T, q) = (0, 0)$  such that

$$\frac{dU^{(1)}}{dT} \Delta T + \frac{dU^{(1)}}{dq} \Delta q > 0,$$

which implies that a self-selecting commitment devices  $(T, q) \gg 0$  increases period-1 intertemporal utility.

## E. Proof of Proposition 3

We can directly prove Proposition 3 from the proof of proposition 1. With the existence of a commitment device, the only changes in the maximization problem are  $\bar{a}_2(S_2)$  and  $\bar{c}_2(S_2)$ . Therefore, we can write the Euler equation in the same way in *Eq. (7)* by replacing  $\bar{a}_2(S_2)$  with  $\bar{a}_2^T(S_2)$ , where  $\bar{a}_2^T(S_2)$  represents the addictive-good response functions in the presence of a tax policy  $(T, q)$  in period 2. The following is the Euler equation with a self-selecting commitment device:

$$p_1 \times u'(c_1) = \underbrace{v_1(a_1, S_1) + \beta \delta v_2(\bar{a}_2^T(S_2), S_2) (1 - d) + \beta \delta^2 v_2(\bar{a}_3(S_2), S_3) (1 - d)}_{\text{Marginal utility}} \quad (58)$$

$$- \underbrace{\left\{ -\beta \delta^2 (1 - d)^2 v_2(a_3, \bar{a}_3(S_2)) (1 - \beta) \frac{\partial \bar{a}_2^T(S_2)}{\partial S_2} \right\}}_{\text{Self-restraint incentive}}$$

From *Eq. (58)*, we know that the conditions for the self-restraint incentive to be positive are

$$\beta < 1 \text{ and } \frac{\partial \bar{a}_2^T(S_2)}{\partial S_2} > 0.$$

Now, we need to check the condition for  $\partial \bar{a}_2^T(S_2) / \partial S_2 > 0$ . The maximization problem in period 2 is

$$\max_{a_2^T, c_2} v(a_2^T, S_2) + u(c_2) + \beta \delta \{v(a_3(S_3), S_3) + u(c_3(S_3))\}, \quad (59)$$

subject to

$$c_2 + p_2(1 + T)a_2^T = y_2 + X. \quad (60)$$

Thus, from *Eqs. (59)* and *(60)*, we have

$$v_1(a_2^T, S_2) + \beta \delta v_2(a_3, S_3) (1 - d) = u'(c_2) p_2 (1 + T). \quad (61)$$



Implicitly differentiating Eq. (61) with respect to  $S_2$ , we have

$$\begin{aligned} & v_{11}(a_2^T, S_2)da_2^T + v_{12}(a_2^T, S_2)dS_2 \\ & + \beta\delta v_{12}(a_3, S_3)da_3 + \beta\delta v_{22}(a_3, S_3)dS_3 \\ = & u''(c_2)p_2(1+T)dc_2. \end{aligned} \tag{62}$$

From the period-2 budget constraint, we have

$$dc_2 + p_2(1+T)da_2 = p_2Tda_2 \rightarrow dc_2 = p_2da_2, \tag{63}$$

From Eqs. (62) and (63), we have

$$\begin{aligned} & v_{11}(a_2^T, S_2)da_2 + u''(c_2)p_2^2(1+T)da_2 + v_{12}(a_2, S_2)dS_2 \\ & + \beta\delta v_{12}(a_3, S_3)da_3 + \beta\delta v_{22}(a_3, S_3)dS_3 = 0. \end{aligned}$$

which is equivalent to

$$\frac{d\bar{a}_2^T}{dS_2} = -\frac{v_{12}(a_2^T, S_2) + \beta\delta v_{12}(a_3, S_3)\bar{a}'_3(S_3)(1-d) + \beta\delta v_{22}(a_3, S_3)(1-d)}{v_{11}(a_2^T, S_2) + u''(c_2)(1+T)p_2^2}. \tag{64}$$

From Eq. (23), we know that  $\frac{\partial \bar{a}_2^T(S_2)}{\partial S_2} > 0$  if  $v_{22}(a_3, S_3) = 0$ .

## References

- Amador, M., I. Werning, and G.-M. Angeletos (2006). Commitment vs. flexibility. *Econometrica* 74(2), 365–396.
- Becker, G. S. and K. M. Murphy (1988). A theory of rational addiction. *Journal of Political Economy* 96(4), 675–700.
- Bénabou, R. and J. Tirole (2002). Self-confidence and personal motivation. *Quarterly Journal of Economics* 117(3), 871–915.
- Bénabou, R. and J. Tirole (2004). Willpower and personal rules. *Journal of Political Economy* 112(4), 848–886.
- Bernheim, B. D., J. Meer, and N. K. Navarro (2016). Do consumers exploit commitment opportunities? evidence from natural experiments involving liquor consumption. *American Economic Journal: Economic Policy* 8(4), 41–69.

- Bond, P. and G. Sigurdsson (2018). Commitment contracts. *Review of Economic Studies* 85(1), 194–222.
- Bradford, D., C. Courtemanche, G. Heutel, P. McAlvanah, and C. Ruhm (2017). Time preferences and consumer behavior. *Journal of Risk and Uncertainty* 55(2), 119–145.
- Bryan, G., D. Karlan, and S. Nelson (2010). Commitment devices. *Annu. Rev. Econ.* 2(1), 671–698.
- DellaVigna, S. and U. Malmendier (2006). Paying not to go to the gym. *American Economic Review* 96(3), 694–719.
- Fang, H. and D. Silverman (2009). Time-inconsistency and welfare program participation: Evidence from the nlsy. *International Economic Review* 50(4), 1043–1077.
- Gruber, J. and B. Köszegi (2001). Is addiction "rational"? Theory and evidence. *Quarterly Journal of Economics* 116(4), 1261–1303.
- Gruber, J. and B. Köszegi (2004). Tax incidence when individuals are time-inconsistent: the case of cigarette excise taxes. *Journal of Public Economics* 88(9-10), 1959–1987.
- Gruber, J. and S. Mullainathan (2006). Do cigarette taxes make smokers happier? In *Happiness and Public Policy*, pp. 109–146. Springer.
- Hinnosaar, M. (2016). Time inconsistency and alcohol sales restrictions. *European Economic Review* 87, 108–131.
- Kang, M. (2020). Demand deposit contracts and bank runs with present biased preferences. *Journal of Banking & Finance* 119, 105901.
- Kang, M. (2021). Aggregate savings under quasi-hyperbolic versus exponential discounting. *Economics Letters* 207, 110006.
- Kocherlakota, N. R. (2001). Looking for evidence of time-inconsistent preferences in asset market data. *Federal Reserve Bank of Minneapolis Quarterly Review* 25(3), 13–24.
- Laibson, D. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics* 112(2), 443–477.
- O’Donoghue, T. and M. Rabin (2001). Choice and procrastination. *Quarterly Journal of Economics* 116(1), 121–160.

- O'Donoghue, T. and M. Rabin (2006). Optimal sin taxes. *Journal of Public Economics* 90(10-11), 1825–1849.
- Phelps, E. S. and R. A. Pollak (1968). On second-best national saving and game-equilibrium growth. *Review of Economic Studies* 35(2), 185–199.
- Pollak, R. A. (1968). Consistent planning. *Review of Economic Studies* 35(2), 201–208.
- Schilbach, F. (2019). Alcohol and self-control: A field experiment in india. *American Economic Review* 109(4), 1290–1322.
- Schleich, J., X. Gassmann, T. Meissner, and C. Faure (2019). A large-scale test of the effects of time discounting, risk aversion, loss aversion, and present bias on household adoption of energy-efficient technologies. *Energy Economics* 80, 377–393.
- Strotz, R. H. (1956). Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies* 23(3), 165–180.
- Werthschulte, M. and A. Löschel (2021). On the role of present bias and biased price beliefs in household energy consumption. *Journal of Environmental Economics and Management* 109, 102500.